OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS   **4753/1**

Methods for Advanced Mathematics (C3)

| Wednesday | **25 MAY 2005** | Afternoon | 1 hour 30 minutes |

Additional materials:
    Answer booklet
    Graph paper
    MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $|3x + 2| = 1$. [3]

2 Given that $\arcsin x = \frac{1}{6}\pi$, find x. Find $\arccos x$ in terms of π. [3]

3 The functions $f(x)$ and $g(x)$ are defined for the domain $x > 0$ as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function $fg(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = f(x)$ onto the curve $y = fg(x)$. [3]

4 The temperature $T\,°C$ of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}, \quad \text{for } t \geqslant 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

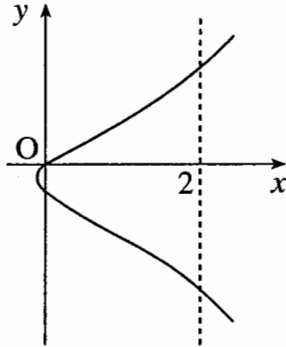Find the time at which the temperature is $40\,°C$. [6]

5 Using the substitution $u = 2x + 1$, show that $\displaystyle\int_0^1 \frac{x}{2x+1}\,dx = \frac{1}{4}(2 - \ln 3)$. [6]

6 A curve has equation $y = \dfrac{x}{2 + 3\ln x}$. Find $\dfrac{dy}{dx}$. Hence find the exact coordinates of the stationary point of the curve. [7]

7   Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\dfrac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points.

[8]

Section B (36 marks)

8   Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

Fig. 8

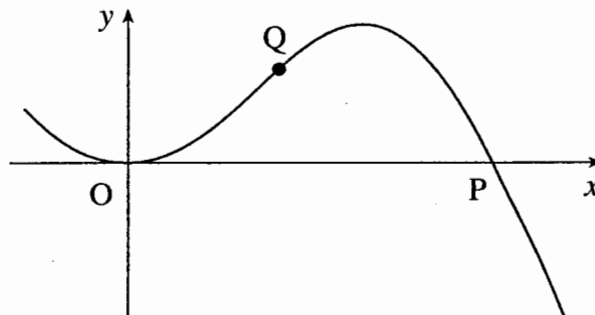(i) Find the x-coordinate of P.   [3]

(ii) Show that Q lies on the line $y = x$.   [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q.   [6]

(iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$.   [7]

[Turn over

9 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leqslant x \leqslant 3$.

Fig. 9 shows the graph of $y = f(x)$.

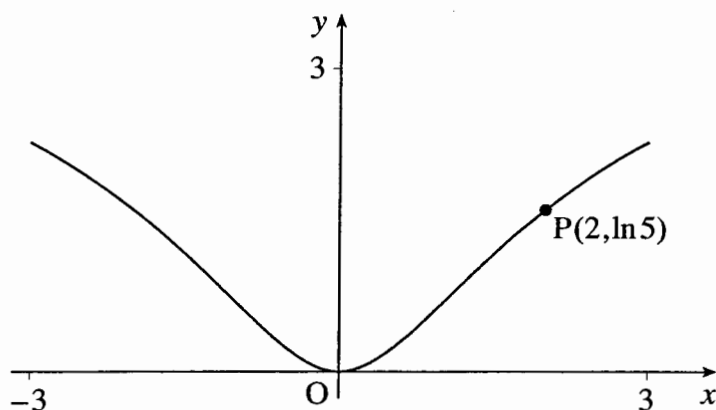**Fig. 9**

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point $P(2, \ln 5)$. [4]

(iii) Explain why the function does not have an inverse for the domain $-3 \leqslant x \leqslant 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leqslant x \leqslant 3$. The inverse of $f(x)$ is the function $g(x)$.

(iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

(v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part **(ii)**. [5]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS   **4753/1**

Methods for Advanced Mathematics (C3)

Wednesday   **18 JANUARY 2006**   Afternoon   1 hour 30 minutes

Additional materials:
    8 page answer booklet
    Graph paper
    MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

•   Write your name, centre number and candidate number in the spaces provided on the answer booklet.

•   Answer **all** the questions.

•   You are permitted to use a graphical calculator in this paper.

•   Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

•   The number of marks is given in brackets [ ] at the end of each question or part question.

•   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

•   The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

       Registered Charity 1066969   **[Turn over**

Section A (36 marks)

1 Given that $y = (1+6x)^{\frac{1}{3}}$, show that $\dfrac{dy}{dx} = \dfrac{2}{y^2}$. [4]

2 A population is P million at time t years. P is modelled by the equation

$$P = 5 + a e^{-bt},$$

where a and b are constants.

The population is initially 8 million, and declines to 6 million after 1 year.

  (i) Use this information to calculate the values of a and b, giving b correct to 3 significant figures. [5]

  (ii) What is the long-term population predicted by the model? [1]

3   **(i)** Express $2\ln x + \ln 3$ as a single logarithm. [2]

  (ii) Hence, given that x satisfies the equation

$$2\ln x + \ln 3 = \ln(5x+2),$$

    show that x is a root of the quadratic equation $3x^2 - 5x - 2 = 0$. [2]

  (iii) Solve this quadratic equation, explaining why only one root is a valid solution of

$$2\ln x + \ln 3 = \ln(5x+2).$$ [3]

4   Fig. 4 shows a cone. The angle between the axis and the slant edge is 30°. Water is poured into the cone at a constant rate of $2\,\text{cm}^3$ per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is $V\,\text{cm}^3$.
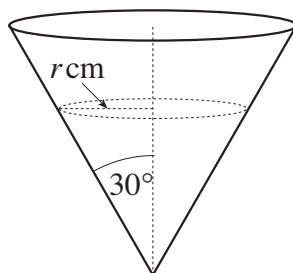

Fig. 4

(i) Write down the value of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$.   [1]

(ii) Show that $V = \dfrac{\sqrt{3}}{3}\pi r^3$, and find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.   [3]

[You may assume that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.]

(iii) Use the results of parts **(i)** and **(ii)** to find the value of $\dfrac{\mathrm{d}r}{\mathrm{d}t}$ when $r = 2$.   [3]

5   A curve is defined implicitly by the equation

$$y^3 = 2xy + x^2.$$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2(x+y)}{3y^2 - 2x}$.   [4]

(ii) Hence write down $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x and y.   [1]

[Turn over

6 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Show that $f^{-1}(x) = \arcsin\left(\dfrac{x-1}{2}\right)$ and state the domain of this function. [4]

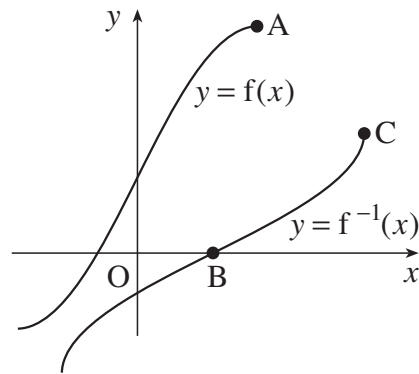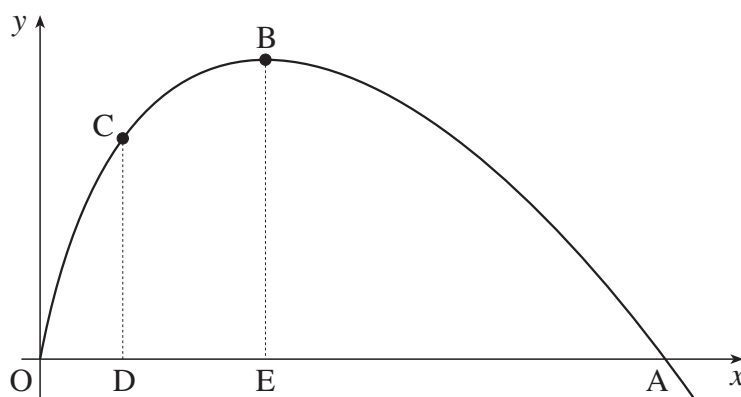Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

Fig. 6

(ii) Write down the coordinates of the points A, B and C. [3]

Section B (36 marks)

7 Fig. 7 shows the curve

$$y = 2x - x \ln x, \text{ where } x > 0.$$

The curve crosses the *x*-axis at A, and has a turning point at B. The point C on the curve has *x*-coordinate 1. Lines CD and BE are drawn parallel to the *y*-axis.

Not to scale

Fig. 7

(i) Find the *x*-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

(iv) Using integration by parts, show that

$$\int x \ln x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 + c.$$

Hence find the exact area of the region enclosed by the curve, the *x*-axis and the lines CD and BE. [7]

[Question 8 is printed overleaf.]

6

8 The function $f(x) = \dfrac{\sin x}{2 - \cos x}$ has domain $-\pi \leqslant x \leqslant \pi$.

Fig. 8 shows the graph of $y = f(x)$ for $0 \leqslant x \leqslant \pi$.
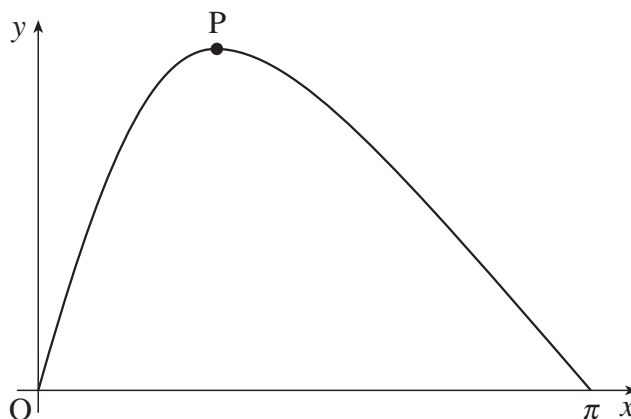
Fig. 8

(i) Find $f(-x)$ in terms of $f(x)$. Hence sketch the graph of $y = f(x)$ for the complete domain $-\pi \leqslant x \leqslant \pi$. [3]

(ii) Show that $f'(x) = \dfrac{2\cos x - 1}{(2 - \cos x)^2}$. Hence find the exact coordinates of the turning point P.

State the range of the function $f(x)$, giving your answer exactly. [8]

(iii) Using the substitution $u = 2 - \cos x$ or otherwise, find the exact value of $\displaystyle\int_0^\pi \dfrac{\sin x}{2 - \cos x}\,dx$. [4]

(iv) Sketch the graph of $y = f(2x)$. [1]

(v) Using your answers to parts (iii) and (iv), write down the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \dfrac{\sin 2x}{2 - \cos 2x}\,dx$. [2]

MEI STRUCTURED MATHEMATICS   **4753/1**

Methods for Advanced Mathematics (C3)

Thursday   **8 JUNE 2006**   Morning   1 hour 30 minutes

Additional materials:
    8 page answer booklet
    Graph paper
    MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.

- Answer **all** the questions.

- You are permitted to use a graphical calculator in this paper.

- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Solve the equation $\left|3x - 2\right| = x$. [3]

2 Show that $\displaystyle\int_0^{\frac{1}{6}\pi} x\sin 2x\, \mathrm{d}x = \dfrac{3\sqrt{3} - \pi}{24}$. [6]

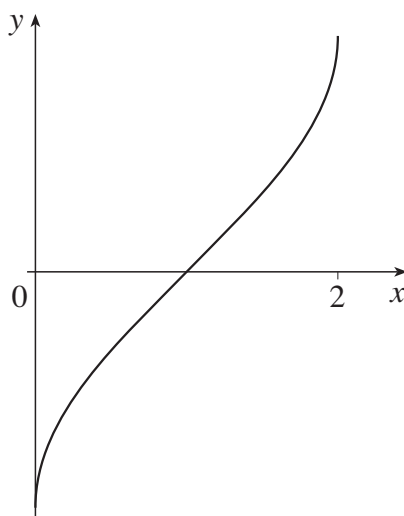**3** Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leqslant x \leqslant 2$.

Fig. 3

  (i) Find x in terms of y, and show that $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \cos y$. [3]

  (ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. [4]

4 Fig. 4 is a diagram of a garden pond.
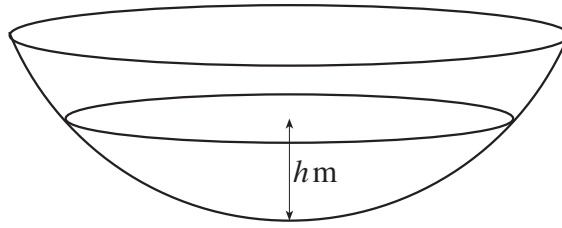
Fig. 4

The volume $V \text{m}^3$ of water in the pond when the depth is h metres is given by

$$V = \tfrac{1}{3}\pi h^2(3 - h).$$

(i) Find $\dfrac{dV}{dh}$. [2]

Water is poured into the pond at the rate of 0.02 m^3 per minute.

(ii) Find the value of $\dfrac{dh}{dt}$ when $h = 0.4$. [4]

5 Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t, t^2 - 1$ and $t^2 + 1$. [3]

6 The mass $M \text{kg}$ of a radioactive material is modelled by the equation

$$M = M_0 e^{-kt},$$

where M_0 is the initial mass, t is the time in years, and k is a constant which measures the rate of radioactive decay.

(i) Sketch the graph of M against t. [2]

(ii) For Carbon 14, $k = 0.000\,121$. Verify that after 5730 years the mass M has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

(iii) Show that, in general, the half-life T is given by $T = \dfrac{\ln 2}{k}$. [3]

(iv) Hence find the half-life of Plutonium 239, given that for this material $k = 2.88 \times 10^{-5}$. [1]

[Turn over

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2 + 3}{x - 1}$. It has a minimum at the point P. The line l is an asymptote to the curve.
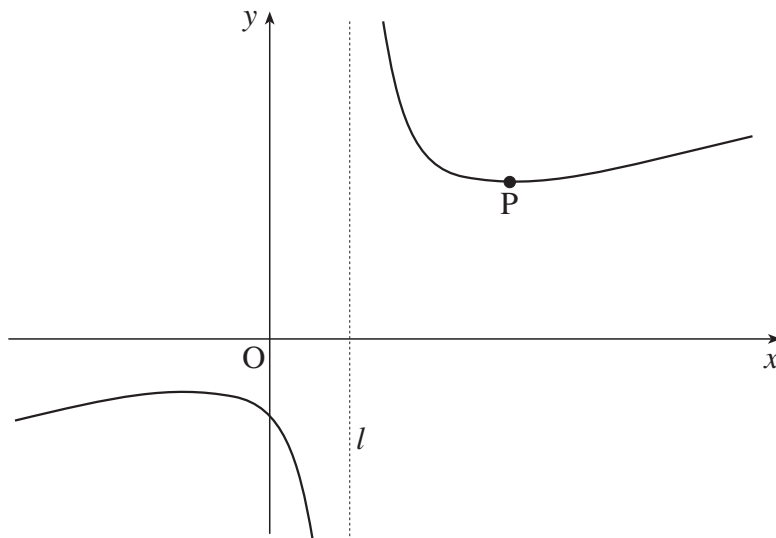

Fig. 7

(i) Write down the equation of the asymptote l. [1]

(ii) Find the coordinates of P. [6]

(iii) Using the substitution $u = x - 1$, show that the area of the region enclosed by the x-axis, the curve and the lines $x = 2$ and $x = 3$ is given by

$$\int_1^2 \left(u + 2 + \frac{4}{u} \right) du.$$

Evaluate this area exactly. [7]

(iv) Another curve is defined by the equation $e^y = \dfrac{x^2 + 3}{x - 1}$. Find $\dfrac{dy}{dx}$ in terms of x and y by differentiating implicitly. Hence find the gradient of this curve at the point where $x = 2$. [4]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{-\frac{1}{5}x}\sin x$, for all x.
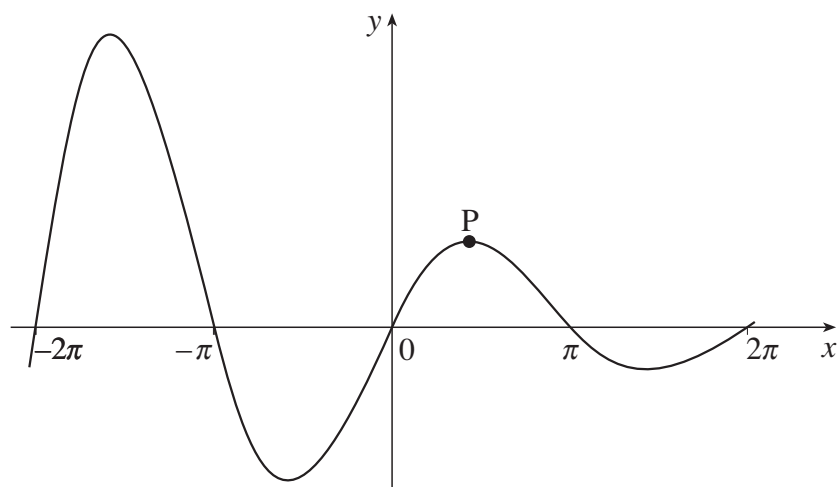
Fig. 8

(i) Sketch the graphs of

(A) $y = \mathrm{f}(2x)$,

(B) $y = \mathrm{f}(x + \pi)$. [4]

(ii) Show that the x-coordinate of the turning point P satisfies the equation $\tan x = 5$.

Hence find the coordinates of P. [6]

(iii) Show that $\mathrm{f}(x + \pi) = -\mathrm{e}^{-\frac{1}{5}\pi}\mathrm{f}(x)$. Hence, using the substitution $u = x - \pi$, show that

$$\int_{\pi}^{2\pi} \mathrm{f}(x)\,\mathrm{d}x = -\mathrm{e}^{-\frac{1}{5}\pi}\int_{0}^{\pi} \mathrm{f}(u)\,\mathrm{d}u.$$

Interpret this result graphically. [You should *not* attempt to integrate $\mathrm{f}(x)$.] [8]

ADVANCED GCE UNIT

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

THURSDAY 18 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional materials:
 Answer booklet (8 pages)
 Graph paper
 MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.

- Answer **all** the questions.

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INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.

- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.

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This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1   Fig.1 shows the graphs of $y = |x|$ and $y = |x - 2| + 1$. The point P is the minimum point of $y = |x - 2| + 1$, and Q is the point of intersection of the two graphs.
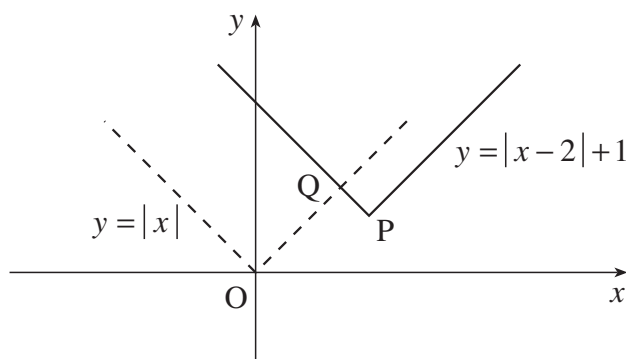
Fig. 1

    (i) Write down the coordinates of P. [1]

    (ii) Verify that the y-coordinate of Q is $1\frac{1}{2}$. [4]

2   Evaluate $\displaystyle\int_1^2 x^2 \ln x \, dx$, giving your answer in an exact form. [5]

3   The value £V of a car is modelled by the equation $V = Ae^{-kt}$, where t is the age of the car in years and A and k are constants. Its value when new is £10 000, and after 3 years its value is £6000.

    (i) Find the values of A and k. [5]

    (ii) Find the age of the car when its value is £2000. [2]

4   Use the method of exhaustion to prove the following result.

              No 1- or 2-digit perfect square ends in 2, 3, 7 or 8

State a generalisation of this result. [3]

5   The equation of a curve is $y = \dfrac{x^2}{2x + 1}$.

    (i) Show that $\dfrac{dy}{dx} = \dfrac{2x(x + 1)}{(2x + 1)^2}$. [4]

    (ii) Find the coordinates of the stationary points of the curve. You need not determine their nature. [4]

6 Fig. 6 shows the triangle OAP, where O is the origin and A is the point $(0, 3)$. The point $P(x, 0)$ moves on the positive x-axis. The point $Q(0, y)$ moves between O and A in such a way that $AQ + AP = 6$.
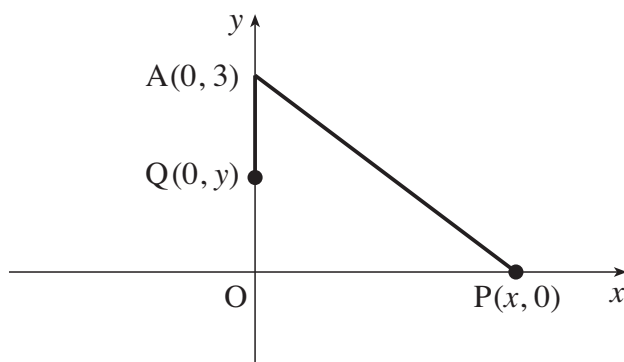
Fig. 6

(i) Write down the length AQ in terms of y. Hence find AP in terms of y, and show that

$$(y + 3)^2 = x^2 + 9.$$
[3]

(ii) Use this result to show that $\dfrac{dy}{dx} = \dfrac{x}{y + 3}$.
[2]

(iii) When $x = 4$ and $y = 2$, $\dfrac{dx}{dt} = 2$. Calculate $\dfrac{dy}{dt}$ at this time.
[3]

Section B (36 marks)

7   Fig. 7 shows part of the curve $y = f(x)$, where $f(x) = x\sqrt{1+x}$. The curve meets the x-axis at the origin and at the point P.
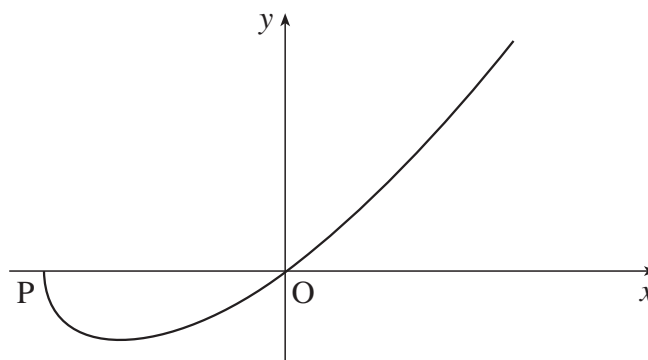
Fig. 7

(i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $f(x)$.
[2]

(ii) Show that $\dfrac{dy}{dx} = \dfrac{2+3x}{2\sqrt{1+x}}$.
[4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function.
[4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\, dx = \int_{0}^{1} \left(u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du.$$

Hence find the area of the region enclosed by the curve and the x-axis.
[8]

8 Fig. 8 shows part of the curve $y = f(x)$, where

$$f(x) = (e^x - 1)^2 \quad \text{for } x \geqslant 0.$$
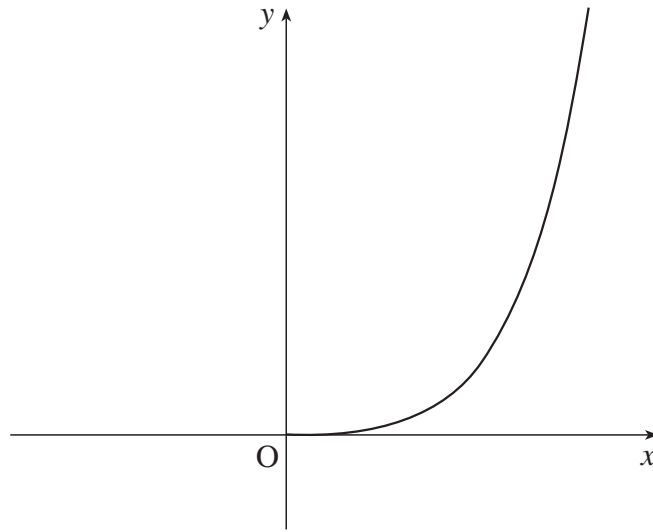
Fig. 8

(i) Find $f'(x)$, and hence calculate the gradient of the curve $y = f(x)$ at the origin and at the point $(\ln 2, 1)$. [5]

The function $g(x)$ is defined by $g(x) = \ln(1 + \sqrt{x})$ for $x \geqslant 0$.

(ii) Show that $f(x)$ and $g(x)$ are inverse functions. Hence sketch the graph of $y = g(x)$.

Write down the gradient of the curve $y = g(x)$ at the point $(1, \ln 2)$. [5]

(iii) Show that $\displaystyle \int (e^x - 1)^2 \, dx = \tfrac{1}{2} e^{2x} - 2e^x + x + c$.

Hence evaluate $\displaystyle \int_0^{\ln 2} (e^x - 1)^2 \, dx$, giving your answer in an exact form. [5]

(iv) Using your answer to part **(iii)**, calculate the area of the region enclosed by the curve $y = g(x)$, the x-axis and the line $x = 1$. [3]

ADVANCED GCE UNIT   **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 11 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
      Answer booklet (8 pages)
      Graph paper
      MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.

- Answer **all** the questions.

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This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Differentiate $\sqrt{1+2x}$. [3]

   (ii) Show that the derivative of $\ln(1 - e^{-x})$ is $\dfrac{1}{e^x - 1}$. [4]

2 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

   On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

3 A curve has equation $2y^2 + y = 9x^2 + 1$.

   (i) Find $\dfrac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at the point A $(1, 2)$. [4]

   (ii) Find the coordinates of the points on the curve at which $\dfrac{dy}{dx} = 0$. [4]

4 A cup of water is cooling. Its initial temperature is 100°C. After 3 minutes, its temperature is 80°C.

   (i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in °C, t is the time in minutes and a and k are constants, find the values of a and k. [5]

   (ii) What is the temperature of the water

      (*A*) after 5 minutes,

      (*B*) in the long term? [3]

5 Prove that the following statement is false.

      For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

6 Fig. 6 shows the curve $y = f(x)$, where $f(x) = \frac{1}{2}\arctan x$.
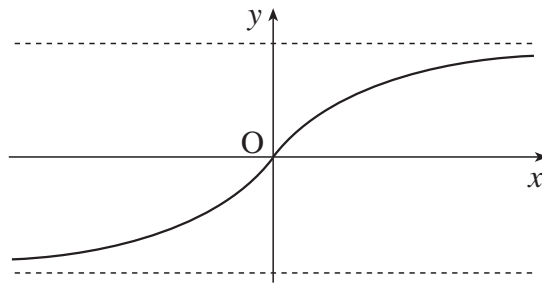
Fig. 6

    (i) Find the range of the function $f(x)$, giving your answer in terms of π.   [2]

    (ii) Find the inverse function $f^{-1}(x)$. Find the gradient of the curve $y = f^{-1}(x)$ at the origin. [5]

    (iii) Hence write down the gradient of $y = \frac{1}{2}\arctan x$ at the origin.   [1]

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2}{1 + 2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.
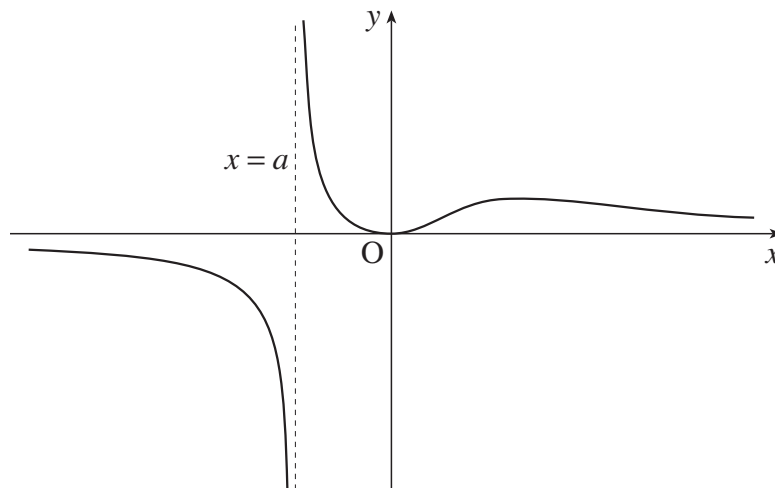
Fig. 7

    (i) Calculate the value of a, giving your answer correct to 3 significant figures.   [3]

    (ii) Show that $\dfrac{dy}{dx} = \dfrac{2x - 2x^4}{(1 + 2x^3)^2}$. Hence determine the coordinates of the turning points of the curve.   [8]

    (iii) Show that the area of the region between the curve and the x-axis from $x = 0$ to $x = 1$ is $\frac{1}{6}\ln 3$.   [5]

8 Fig. 8 shows part of the curve $y = x \cos 2x$, together with a point P at which the curve crosses the x-axis.
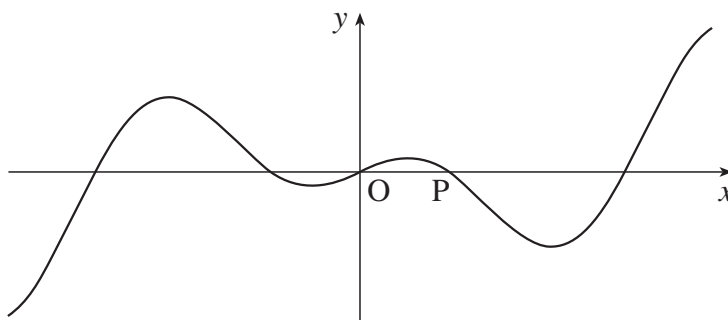
Fig. 8

 (i) Find the exact coordinates of P. [3]

 (ii) Show algebraically that $x \cos 2x$ is an odd function, and interpret this result graphically. [3]

 (iii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

 (iv) Show that turning points occur on the curve for values of x which satisfy the equation $x \tan 2x = \tfrac{1}{2}$. [2]

 (v) Find the gradient of the curve at the origin.

 Show that the second derivative of $x \cos 2x$ is zero when $x = 0$. [4]

 (vi) Evaluate $\displaystyle\int_0^{\frac{1}{4}\pi} x \cos 2x \, \mathrm{d}x$, giving your answer in terms of π. Interpret this result graphically. [6]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008

Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
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This document consists of **4** printed pages.

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Section A (36 marks)

1   Solve the inequality $|2x - 1| \leqslant 3$. [4]

2   Find $\displaystyle\int x e^{3x}\, dx$. [4]

3   (i) State the algebraic condition for the function $f(x)$ to be an even function.

What geometrical property does the graph of an even function have? [2]

(ii) State whether the following functions are odd, even or neither.

(A) $f(x) = x^2 - 3$

(B) $g(x) = \sin x + \cos x$

(C) $h(x) = \dfrac{1}{x + x^3}$ [3]

4   Show that $\displaystyle\int_1^4 \dfrac{x}{x^2 + 2}\, dx = \tfrac{1}{2}\ln 6$. [4]

5   Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \dfrac{1}{\sqrt{e}}$. [6]

6   In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30e^{-0.1t}.$$

(i) Find the initial mass of the chemical.

What is the mass of chemical in the long term? [3]

(ii) Find the time when the mass is 30 grams. [3]

(iii) Sketch the graph of m against t. [2]

7   Given that $x^2 + xy + y^2 = 12$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]

Section B (36 marks)

8   Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{1 + \cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

Fig. 8

   (i) Find the y-coordinate of P. [1]

   (ii) Find $f'(x)$. Hence find the gradient of the curve at the point P. [5]

   (iii) Show that the derivative of $\dfrac{\sin x}{1 + \cos x}$ is $\dfrac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

   (iv) Show that $f^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function f(x) is defined by f(x) = $\sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

  (i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P (a, b) on the semicircle. The tangent at P is shown.

Fig. 9

  (ii) (*A*) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

  (*B*) Differentiate $\sqrt{4 - x^2}$ and deduce the value of f$'(a)$.

  (*C*) Show that your answers to parts (*A*) and (*B*) are equivalent. [6]

The function g(x) is defined by g(x) = 3f($x - 2$), for $0 \leqslant x \leqslant 4$.

  (iii) Describe a sequence of two transformations that would map the curve $y = $ f(x) onto the curve $y = $ g(x).

  Hence sketch the curve $y = $ g(x). [6]

  (iv) Show that if $y = $ g(x) then $9x^2 + y^2 = 36x$. [3]

ADVANCED GCE   **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008   Morning

Time: 1 hour 30 minutes

Additional materials (enclosed):   None

Additional materials (required):
     Answer Booklet (8 pages)
     Graph paper
     MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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Section A (36 marks)

1   Solve the inequality $|2x - 1| \leqslant 3$. [4]

2   Find $\displaystyle\int xe^{3x}\, dx$. [4]

3   **(i)** State the algebraic condition for the function $f(x)$ to be an even function.

   What geometrical property does the graph of an even function have? [2]

   (ii) State whether the following functions are odd, even or neither.

   (A) $f(x) = x^2 - 3$

   (B) $g(x) = \sin x + \cos x$

   (C) $h(x) = \dfrac{1}{x + x^3}$ [3]

4   Show that $\displaystyle\int_1^4 \frac{x}{x^2 + 2}\, dx = \tfrac{1}{2}\ln 6$. [4]

5   Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \dfrac{1}{\sqrt{e}}$. [6]

6   In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30e^{-0.1t}.$$

   (i) Find the initial mass of the chemical.

   What is the mass of chemical in the long term? [3]

   (ii) Find the time when the mass is 30 grams. [3]

   (iii) Sketch the graph of m against t. [2]

7   Given that $x^2 + xy + y^2 = 12$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]

Section B (36 marks)

8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{1 + \cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

Fig. 8

    (i) Find the y-coordinate of P. [1]

    (ii) Find $f'(x)$. Hence find the gradient of the curve at the point P. [5]

    (iii) Show that the derivative of $\dfrac{\sin x}{1 + \cos x}$ is $\dfrac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

    (iv) Show that $f^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function f(x) is defined by f(x) = $\sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

    (i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P (a, b) on the semicircle. The tangent at P is shown.

Fig. 9

    (ii) (*A*) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

        (*B*) Differentiate $\sqrt{4 - x^2}$ and deduce the value of f$'(a)$.

        (*C*) Show that your answers to parts (*A*) and (*B*) are equivalent. [6]

The function g(x) is defined by g(x) = 3f($x - 2$), for $0 \leqslant x \leqslant 4$.

    (iii) Describe a sequence of two transformations that would map the curve $y = $ f(x) onto the curve $y = $ g(x).

        Hence sketch the curve $y = $ g(x). [6]

    (iv) Show that if $y = $ g(x) then $9x^2 + y^2 = 36x$. [3]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
* 8 page Answer Booklet
* Graph paper
* MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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* Use black ink. Pencil may be used for graphs and diagrams only.
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* Answer **all** the questions.
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* This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1   Solve the inequality $|x - 1| < 3$. **[3]**

2   **(i)** Differentiate $x \cos 2x$ with respect to x. **[3]**

   (ii) Integrate $x \cos 2x$ with respect to x. **[4]**

3   Given that $f(x) = \frac{1}{2} \ln(x - 1)$ and $g(x) = 1 + e^{2x}$, show that $g(x)$ is the inverse of $f(x)$. **[3]**

4   Find the exact value of $\displaystyle\int_0^2 \sqrt{1 + 4x}\, dx$, showing your working. **[5]**

5   **(i)** State the period of the function $f(x) = 1 + \cos 2x$, where x is in degrees. **[1]**

   (ii) State a sequence of two geometrical transformations which maps the curve $y = \cos x$ onto the curve $y = f(x)$. **[4]**

   (iii) Sketch the graph of $y = f(x)$ for $-180° < x < 180°$. **[3]**

6   **(i)** Disprove the following statement.

$$\text{'If } p > q, \text{ then } \frac{1}{p} < \frac{1}{q}.\text{'}$$ **[2]**

   (ii) State a condition on p and q so that the statement is true. **[1]**

7   The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$.

   (i) Show that $\dfrac{dy}{dx} = -\left(\dfrac{y}{x}\right)^{\frac{1}{3}}$. **[4]**

Both x and y are functions of t.

   (ii) Find the value of $\dfrac{dy}{dt}$ when $x = 1$, $y = 8$ and $\dfrac{dx}{dt} = 6$. **[3]**

Section B (36 marks)

8   Fig. 8 shows the curve $y = x^2 - \frac{1}{8} \ln x$. P is the point on this curve with x-coordinate 1, and R is the point $\left(0, -\frac{7}{8}\right)$.
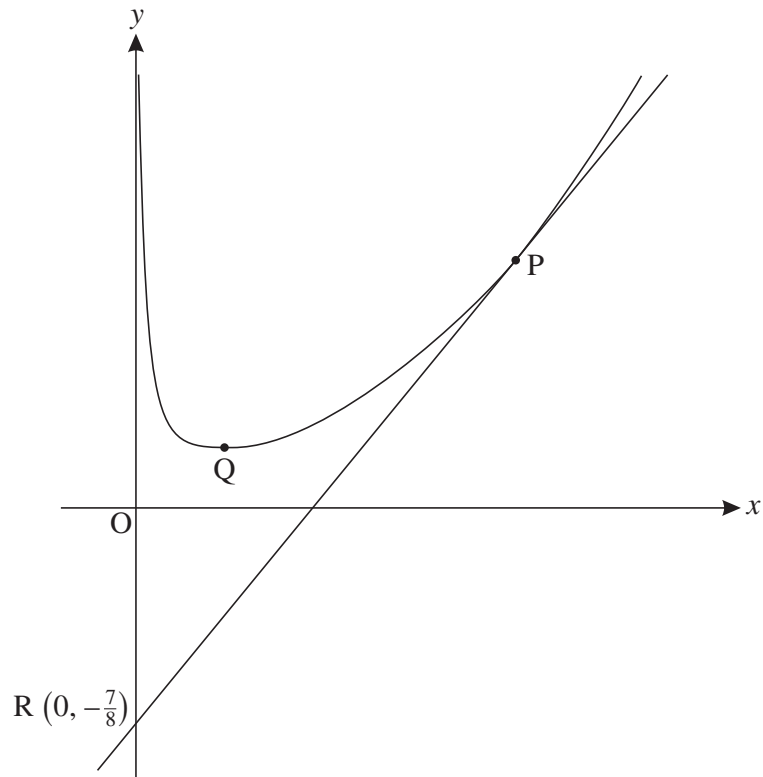
Fig. 8

(i)   Find the gradient of PR. **[3]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Hence show that PR is a tangent to the curve. **[3]**

(iii) Find the exact coordinates of the turning point Q. **[5]**

(iv) Differentiate $x \ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8} \ln x$, the x-axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4} \ln 2$. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.
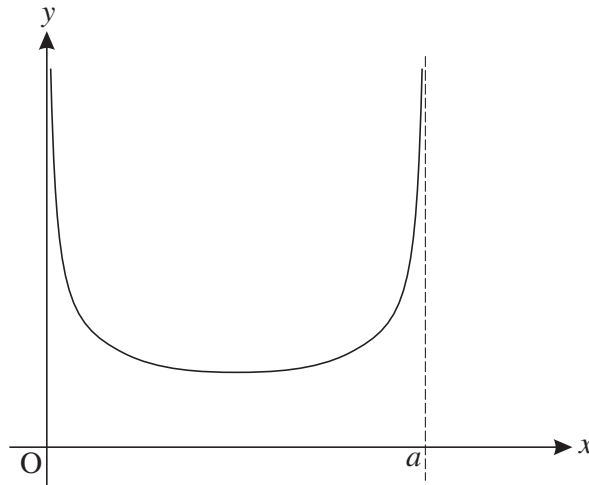
Fig. 9

(i) Find a. Hence write down the domain of the function. **[3]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x - 1}{\left(2x - x^2\right)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function. **[8]**

The function $g(x)$ is defined by $g(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

(iii) (A) Show algebraically that $g(x)$ is an even function.

(B) Show that $g(x - 1) = f(x)$.

(C) Hence prove that the curve $y = f(x)$ is symmetrical, and state its line of symmetry. **[7]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)　　　　　　　　　　　　　　**4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
* 8 page Answer Booklet
* Graph paper
* MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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* Use black ink. Pencil may be used for graphs and diagrams only.
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INFORMATION FOR CANDIDATES

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Turn over

Section A (36 marks)

1 Evaluate $\displaystyle\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. **[3]**

2 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

   (i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k. **[5]**

   (ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. **[3]**

3 Sketch the curve $y = 2\arccos x$ for $-1 \leqslant x \leqslant 1$. **[3]**

4 Fig. 4 shows a sketch of the graph of $y = 2|x - 1|$. It meets the x- and y-axes at $(a, 0)$ and $(0, b)$ respectively.
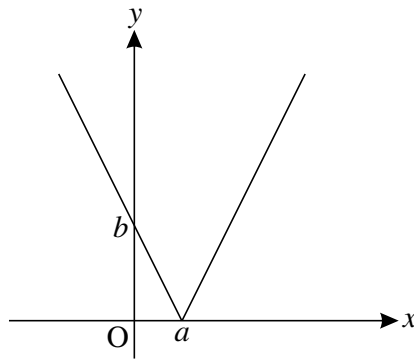
Fig. 4

   Find the values of a and b. **[3]**

5 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

   (i) By differentiating implicitly, find $\dfrac{dy}{dx}$ in terms of x and y. **[3]**

   (ii) Find an expression for y in terms of x, and differentiate it to verify the result in part **(i)**. **[4]**

6 Given that $f(x) = \dfrac{x + 1}{x - 1}$, show that $ff(x) = x$.

   Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? **[5]**

7 **(i)** Show that

(A) $(x - y)(x^2 + xy + y^2) = x^3 - y^3$,

(B) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.
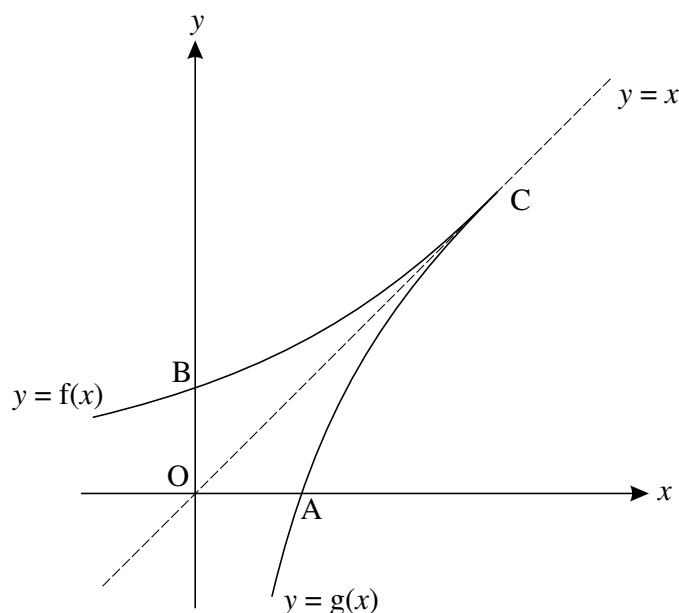
Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are $(1, 1)$. **[5]**

(ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. **[2]**

(iii) Evaluate $\int_0^1 f(x)\, dx$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x\, dx$.

Hence show that $\int_{e^{-1}}^1 g(x)\, dx = \frac{1}{e}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

9 Fig. 9 shows the curve $y = \dfrac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.
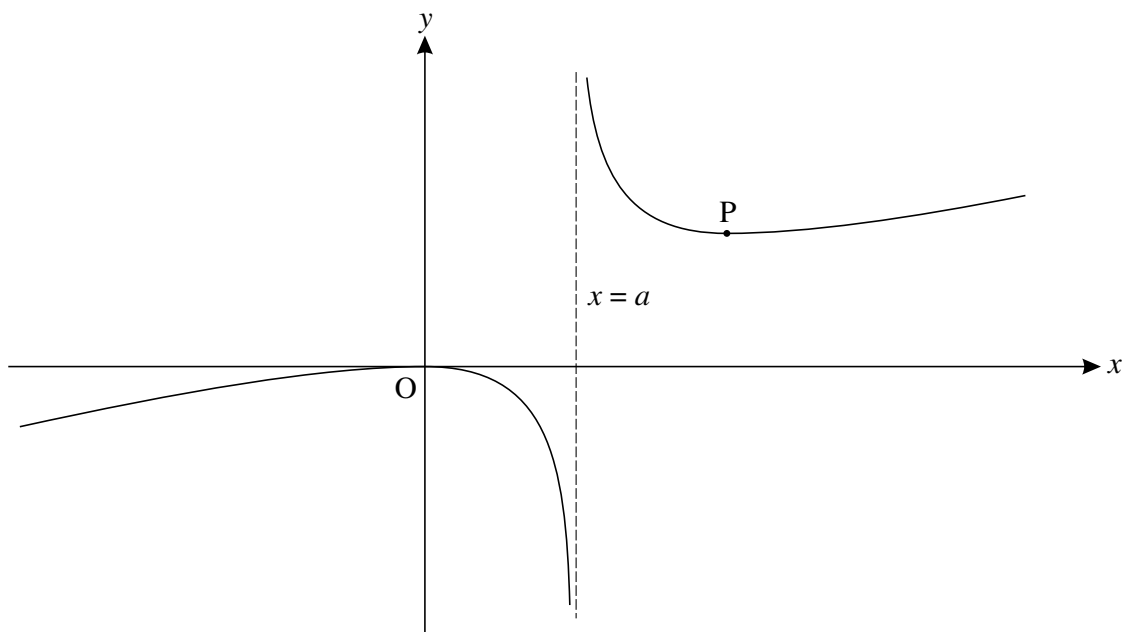
Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x(3x-2)}{(3x-1)^2}$. **[3]**

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. **[7]**

(iv) Using the substitution $u = 3x - 1$, show that $\displaystyle\int \dfrac{x^2}{3x-1}\, dx = \dfrac{1}{27}\int \left(u + 2 + \dfrac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 1$. **[7]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

*4 7 5 3 0 1 *

INSTRUCTIONS TO CANDIDATES

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Turn over

Section A (36 marks)

1 Solve the equation $e^{2x} - 5e^x = 0$. **[4]**

2 The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is $100\,°C$, and after 5 minutes the temperature is $60\,°C$.

    (i) Find b and k. **[4]**

    (ii) Find at what time the temperature reaches $50\,°C$. **[2]**

3   **(i)** Given that $y = \sqrt[3]{1 + 3x^2}$, use the chain rule to find $\dfrac{dy}{dx}$ in terms of x. **[3]**

    (ii) Given that $y^3 = 1 + 3x^2$, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y. Show that this result is equivalent to the result in part **(i)**. **[4]**

4 Evaluate the following integrals, giving your answers in exact form.

    (i) $\displaystyle\int_0^1 \frac{2x}{x^2 + 1}\, dx$. **[3]**

    (ii) $\displaystyle\int_0^1 \frac{2x}{x + 1}\, dx$. **[5]**

5 The curves in parts **(i)** and **(ii)** have equations of the form $y = a + b\sin cx$, where a, b and c are constants. For each curve, find the values of a, b and c.

    (i)
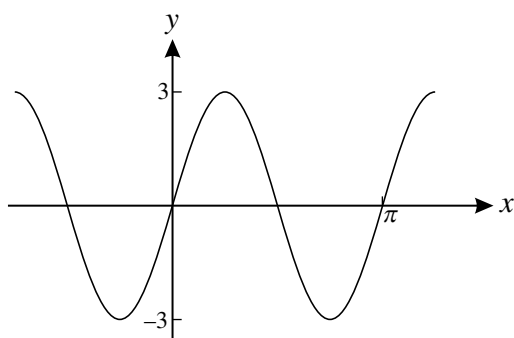
[2]

    (ii)
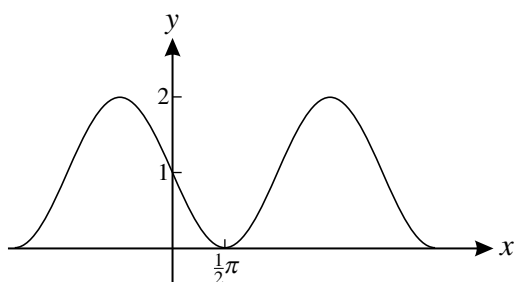
[2]

6   Write down the conditions for f(x) to be an odd function and for g(x) to be an even function.

Hence prove that, if f(x) is odd and g(x) is even, then the composite function gf(x) is even.   **[4]**

7   Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: let $\arcsin x = \theta$.]   **[3]**

Section B (36 marks)

8   Fig. 8 shows part of the curve $y = x \cos 3x$.

The curve crosses the x-axis at O, P and Q.
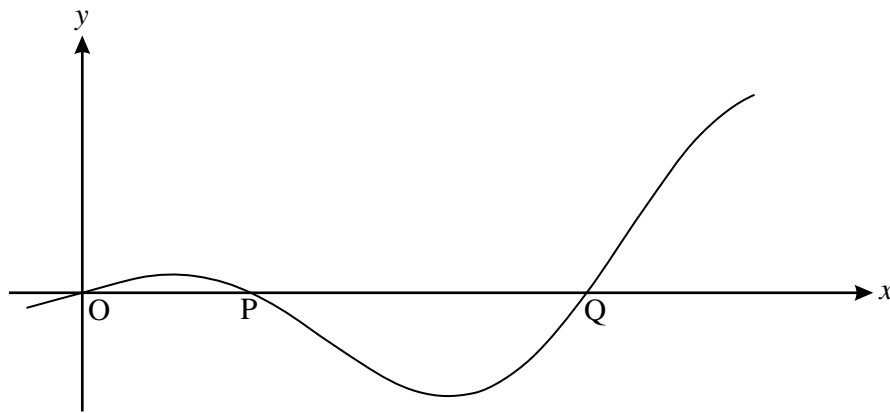
Fig. 8

   (i) Find the exact coordinates of P and Q.   **[4]**

  (ii) Find the exact gradient of the curve at the point P.

Show also that the turning points of the curve occur when $x \tan 3x = \frac{1}{3}$.   **[7]**

 (iii) Find the area of the region enclosed by the curve and the x-axis between O and P, giving your answer in exact form.   **[6]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \dfrac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leqslant x \leqslant 2$.
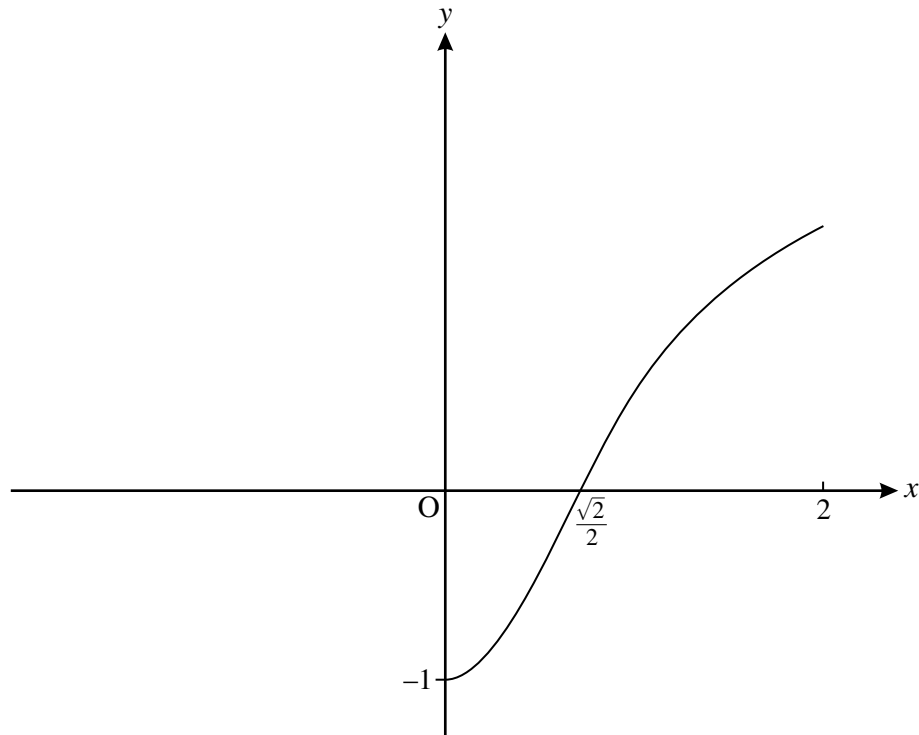
Fig. 9

(i) Show that $f'(x) = \dfrac{6x}{(x^2 + 1)^2}$, and hence that $f(x)$ is an increasing function for $x > 0$. **[5]**

(ii) Find the range of $f(x)$. **[2]**

(iii) **Given** that $f''(x) = \dfrac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $f'(x)$. **[4]**

The function $g(x)$ is the inverse function of $f(x)$.

(iv) Write down the domain and range of $g(x)$. Add a sketch of the curve $y = g(x)$ to a copy of Fig. 9. **[4]**

(v) Show that $g(x) = \sqrt{\dfrac{x + 1}{2 - x}}$. **[4]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)   **4753/01**

Methods for Advanced Mathematics (C3)

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

4 7 5 3 0 1

INSTRUCTIONS TO CANDIDATES

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OCR is an exempt Charity

Turn over

Section A (36 marks)

1 Evaluate $\displaystyle\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$. **[3]**

2 Given that $f(x) = |x|$ and $g(x) = x + 1$, sketch the graphs of the composite functions $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. **[4]**

3 **(i)** Differentiate $\sqrt{1 + 3x^2}$. **[3]**

(ii) Hence show that the derivative of $x\sqrt{1 + 3x^2}$ is $\dfrac{1 + 6x^2}{\sqrt{1 + 3x^2}}$. **[4]**

4 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).
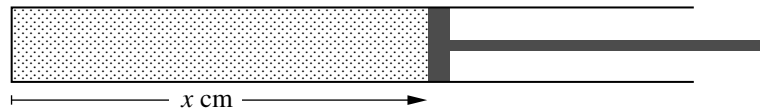
Fig. 4

The pressure of the gas in atmospheric units is given by $p = \dfrac{100}{x}$, where x cm is the distance of the piston from the closed end. At a certain moment, $x = 50$, and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? **[6]**

5 Given that $y^3 = xy - x^2$, show that $\dfrac{dy}{dx} = \dfrac{y - 2x}{3y^2 - x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. **[7]**

6 The function $f(x)$ is defined by

$$f(x) = 1 + 2\sin 3x, \quad -\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. **[6]**

7 State whether the following statements are true or false; if false, provide a counter-example.

(i) If a is rational and b is rational, then $a + b$ is rational.

(ii) If a is rational and b is irrational, then $a + b$ is irrational.

(iii) If a is irrational and b is irrational, then $a + b$ is irrational. **[3]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = 3\ln x + x - x^2$.

The curve crosses the x-axis at P and Q, and has a turning point at R. The x-coordinate of Q is approximately 2.05.
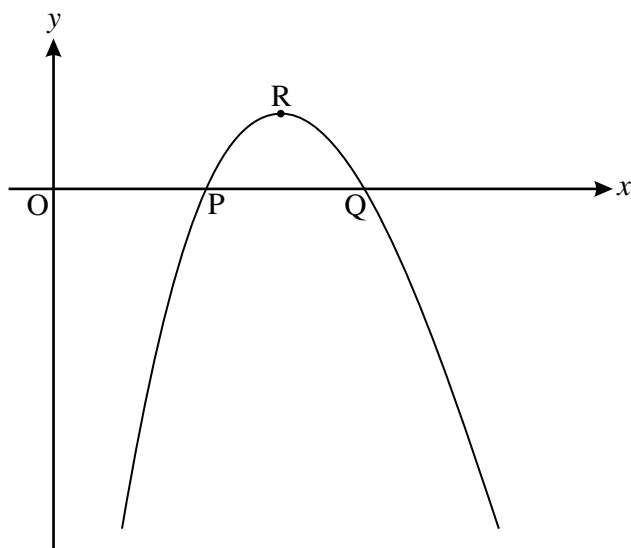
Fig. 8

  (i) Verify that the coordinates of P are (1, 0). **[1]**

 (ii) Find the coordinates of R, giving the y-coordinate correct to 3 significant figures.

     Find $\dfrac{d^2y}{dx^2}$, and use this to verify that R is a maximum point. **[9]**

(iii) Find $\displaystyle\int \ln x \, dx$.

     Hence calculate the area of the region enclosed by the curve and the x-axis between P and Q, giving your answer to 2 significant figures. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \dfrac{e^{2x}}{1 + e^{2x}}$. The curve crosses the y-axis at P.
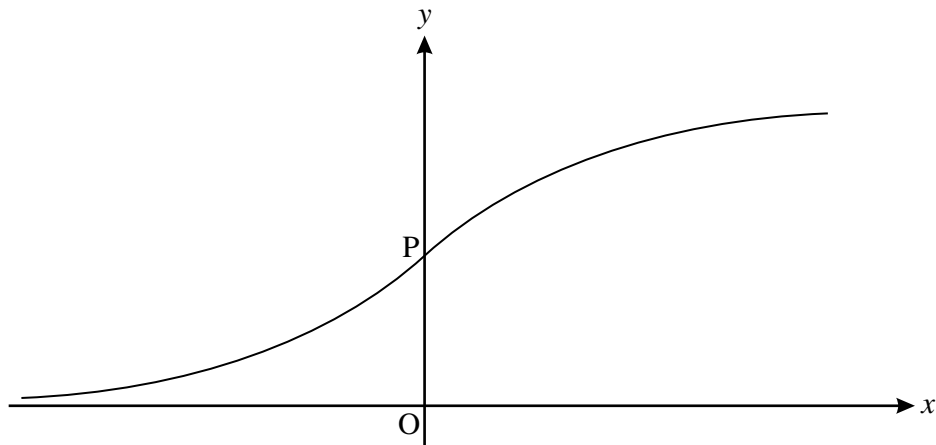
Fig. 9

(i) Find the coordinates of P. **[1]**

(ii) Find $\dfrac{dy}{dx}$, simplifying your answer.

Hence calculate the gradient of the curve at P. **[4]**

(iii) Show that the area of the region enclosed by $y = f(x)$, the x-axis, the y-axis and the line $x = 1$ is $\frac{1}{2}\ln\left(\dfrac{1 + e^2}{2}\right)$. **[5]**

The function $g(x)$ is defined by $g(x) = \frac{1}{2}\left(\dfrac{e^x - e^{-x}}{e^x + e^{-x}}\right)$.

(iv) Prove algebraically that $g(x)$ is an odd function.

Interpret this result graphically. **[3]**

(v) (*A*) Show that $g(x) + \frac{1}{2} = f(x)$.

(*B*) Describe the transformation which maps the curve $y = g(x)$ onto the curve $y = f(x)$.

(*C*) What can you conclude about the symmetry of the curve $y = f(x)$? **[6]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

QUESTION PAPER

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

* The question paper will be found in the centre of the printed answer book.
* Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
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* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a scientific or graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

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* The total number of marks for this paper is **72**.
* The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

* Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1   Given that $y = \sqrt[3]{1 + x^2}$, find $\dfrac{dy}{dx}$. **[4]**

2   Solve the inequality $|2x + 1| \geqslant 4$. **[4]**

3   The area of a circular stain is growing at a rate of $1\,\text{mm}^2$ per second. Find the rate of increase of its radius at an instant when its radius is $2\,\text{mm}$. **[5]**

4   Use the triangle in Fig. 4 to prove that $\sin^2\theta + \cos^2\theta = 1$. For what values of θ is this proof valid? **[3]**
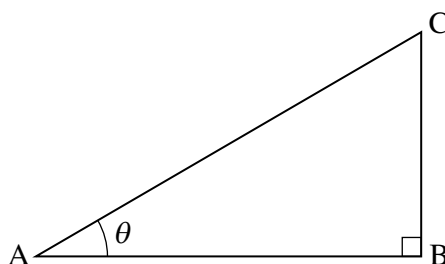
Fig. 4

5   **(i)** On a single set of axes, sketch the curves $y = e^x - 1$ and $y = 2e^{-x}$. **[3]**

   (ii) Find the exact coordinates of the point of intersection of these curves. **[5]**

6   A curve is defined by the equation $(x + y)^2 = 4x$. The point $(1, 1)$ lies on this curve.

   By differentiating implicitly, show that $\dfrac{dy}{dx} = \dfrac{2}{x + y} - 1$.

   Hence verify that the curve has a stationary point at $(1, 1)$. **[4]**

7 Fig. 7 shows the curve $y = f(x)$, where $f(x) = 1 + 2 \arctan x$, $x \in \mathbb{R}$. The scales on the *x*- and *y*-axes are the same.
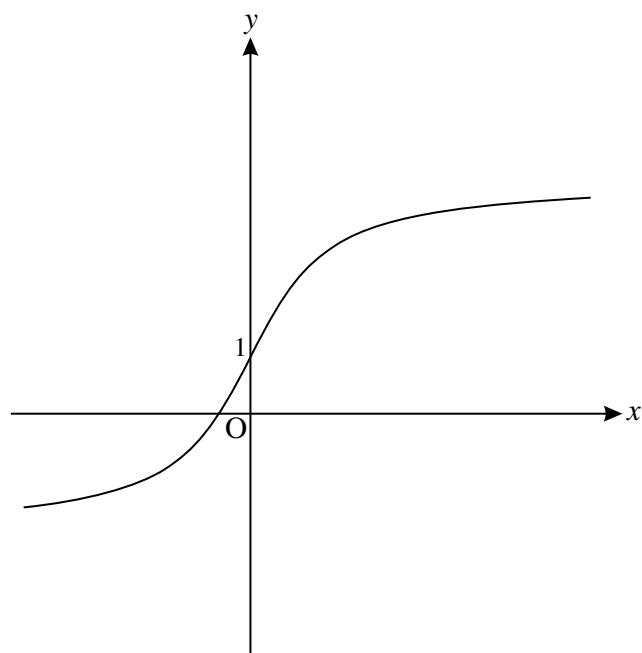
Fig. 7

    (i) Find the range of f, giving your answer in terms of π.   **[3]**

    (ii) Find $f^{-1}(x)$, and add a sketch of the curve $y = f^{-1}(x)$ to the copy of Fig. 7.   **[5]**

4

Section B (36 Marks)

8   **(i)** Use the substitution $u = 1 + x$ to show that

$$\int_0^1 \frac{x^3}{1+x} \, dx = \int_a^b \left(u^2 - 3u + 3 - \frac{1}{u} \right) du,$$

where a and b are to be found.

Hence evaluate $\int_0^1 \frac{x^3}{1+x} \, dx$, giving your answer in exact form.   **[7]**

Fig. 8 shows the curve $y = x^2 \ln(1 + x)$.
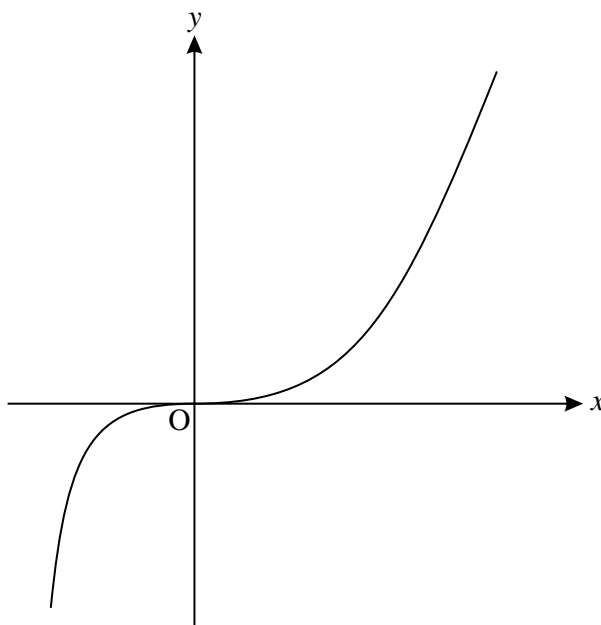
Fig. 8

(ii) Find $\frac{dy}{dx}$.

Verify that the origin is a stationary point of the curve.   **[5]**

(iii) Using integration by parts, and the result of part **(i)**, find the exact area enclosed by the curve $y = x^2 \ln(1 + x)$, the x-axis and the line $x = 1$.   **[6]**

9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{\cos^2 x}$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, together with its asymptotes $x = \frac{1}{2}\pi$ and $x = -\frac{1}{2}\pi$.
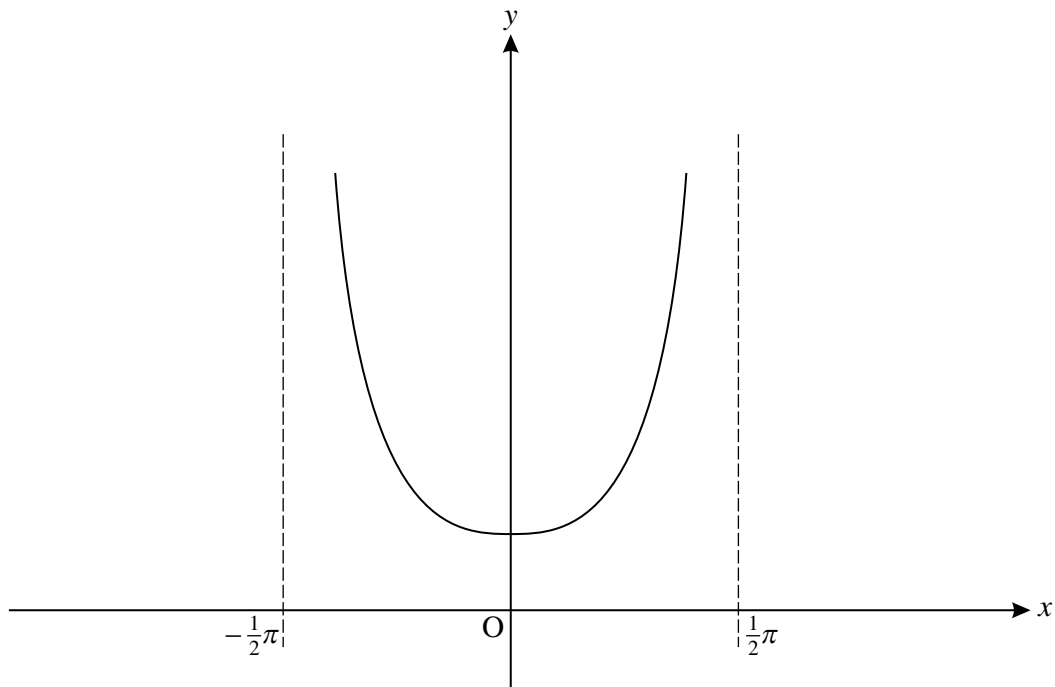
Fig. 9

(i) Use the quotient rule to show that the derivative of $\dfrac{\sin x}{\cos x}$ is $\dfrac{1}{\cos^2 x}$. **[3]**

(ii) Find the area bounded by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = \frac{1}{4}\pi$. **[3]**

The function $g(x)$ is defined by $g(x) = \frac{1}{2}f\left(x + \frac{1}{4}\pi\right)$.

(iii) Verify that the curves $y = f(x)$ and $y = g(x)$ cross at $(0, 1)$. **[3]**

(iv) State a sequence of two transformations such that the curve $y = f(x)$ is mapped to the curve $y = g(x)$.

On the copy of Fig. 9, sketch the curve $y = g(x)$, indicating clearly the coordinates of the minimum point and the equations of the asymptotes to the curve. **[8]**

(v) Use your result from part **(ii)** to write down the area bounded by the curve $y = g(x)$, the x-axis, the y-axis and the line $x = -\frac{1}{4}\pi$. **[1]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)   **4753/01**

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
* Printed answer book 4753/01
* MEI Examination Formulae and Tables (MF2)

Other materials required:
* Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

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* The total number of marks for this paper is **72**.
* The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

* Do not send this question paper for marking; it should be retained in the centre or destroyed.

OCR is an exempt Charity   **Turn over**

Section A (36 marks)

1 Solve the equation $|2x - 1| = |x|$. **[4]**

2 Given that $f(x) = 2 \ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. **[3]**

3 **(i)** Differentiate $\dfrac{\ln x}{x^2}$, simplifying your answer. **[4]**

   (ii) Using integration by parts, show that $\displaystyle\int \dfrac{\ln x}{x^2}\, dx = -\dfrac{1}{x}(1 + \ln x) + c$. **[4]**

4 The height h metres of a tree after t years is modelled by the equation

$$h = a - be^{-kt},$$

where a, b and k are positive constants.

   (i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b. **[3]**

   (ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. **[3]**

5 Given that $y = x^2\sqrt{1 + 4x}$, show that $\dfrac{dy}{dx} = \dfrac{2x(5x + 1)}{\sqrt{1 + 4x}}$. **[5]**

6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.

   (i) Verify that the point $P\left(\tfrac{1}{6}\pi, \tfrac{1}{6}\pi\right)$ lies on the curve. **[1]**

   (ii) Find $\dfrac{dy}{dx}$ in terms of x and y.

      Hence find the gradient of the curve at the point P. **[5]**

7 **(i)** Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

   (ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

Section B (36 marks)

8
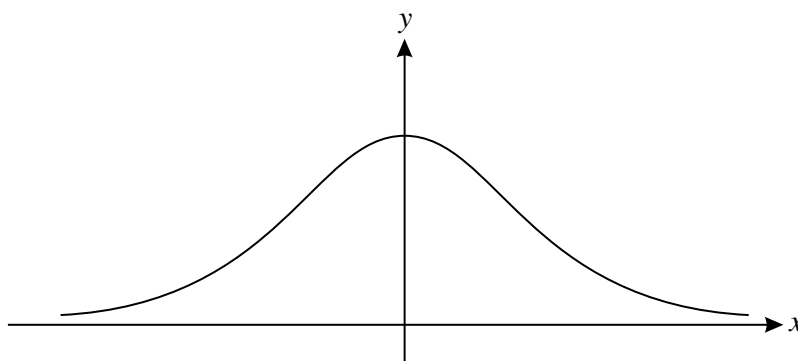
Fig. 8

Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{e^x + e^{-x} + 2}$.

(i) Show algebraically that $f(x)$ is an even function, and state how this property relates to the curve $y = f(x)$. **[3]**

(ii) Find $f'(x)$. **[3]**

(iii) Show that $f(x) = \dfrac{e^x}{(e^x + 1)^2}$. **[2]**

(iv) Hence, using the substitution $u = e^x + 1$, or otherwise, find the exact area enclosed by the curve $y = f(x)$, the x-axis, and the lines $x = 0$ and $x = 1$. **[5]**

(v) Show that there is only one point of intersection of the curves $y = f(x)$ and $y = \frac{1}{4}e^x$, and find its coordinates. **[5]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = f(x)$. The endpoints of the curve are P$(-\pi, 1)$ and Q$(\pi, 3)$, and $f(x) = a + \sin bx$, where a and b are constants.
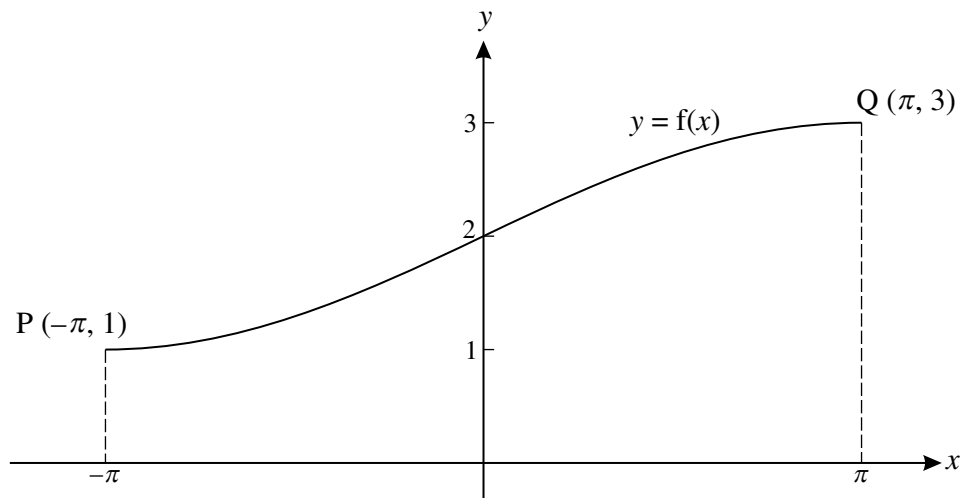
Fig. 9

   (i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. **[3]**

   (ii) Find the gradient of the curve $y = f(x)$ at the point $(0, 2)$.

   Show that there is no point on the curve at which the gradient is greater than this. **[5]**

   (iii) Find $f^{-1}(x)$, and state its domain and range.

   Write down the gradient of $y = f^{-1}(x)$ at the point $(2, 0)$. **[6]**

   (iv) Find the area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = \pi$. **[4]**

OCR
RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

* The Question Paper will be found in the centre of the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

* Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Section A (36 marks)

1   Differentiate $x^2 \tan 2x$. **[3]**

2   The functions f(x) and g(x) are defined as follows.

$$f(x) = \ln x, \qquad x > 0$$
$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. **[4]**

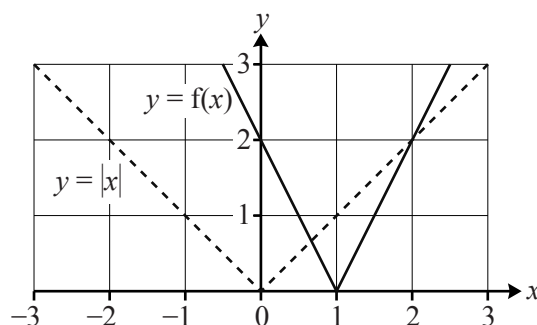3   Show that $\displaystyle\int_0^{\frac{\pi}{2}} x\cos\tfrac{1}{2}x \, dx = \dfrac{\sqrt{2}}{2}\,\pi + 2\sqrt{2} - 4$. **[5]**

4   Prove or disprove the following statement:

‘No cube of an integer has 2 as its units digit.’ **[2]**

5   Each of the graphs of $y = f(x)$ and $y = g(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for f(x) and g(x).

(i)

[3]

(ii)
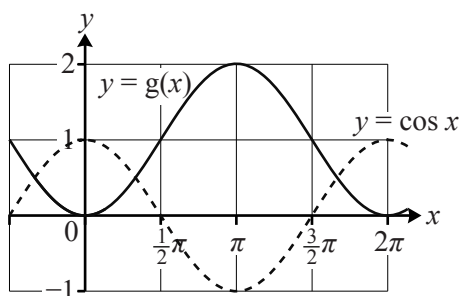
[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

    (i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time.   **[4]**

    (ii) Find the rate at which the area of the slick is increasing when $t = 2$.   **[4]**

7 Fig. 7 shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.
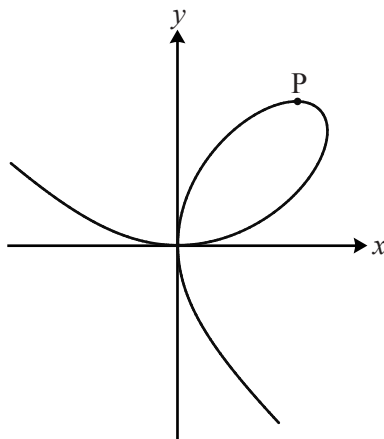
Fig. 7

    (i) Show that $\dfrac{dy}{dx} = \dfrac{y - x^2}{y^2 - x}$.   **[4]**

    (ii) Hence find the exact x-coordinate of P.   **[4]**

Section B (36 marks)

8   Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$.

The curve meets these lines at P and Q respectively. R is the point (11, 11).
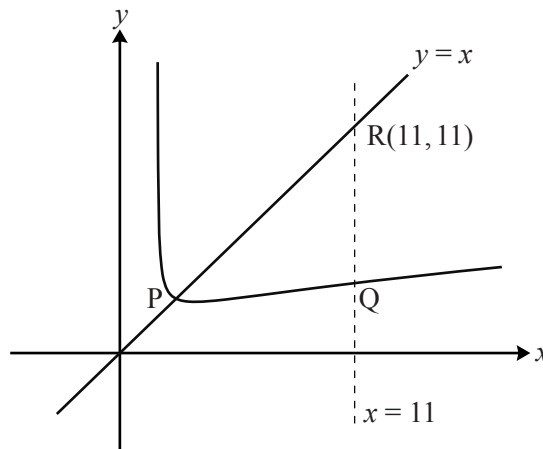
Fig. 8

(i) Verify that the *x*-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\dfrac{dy}{dx} = \dfrac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\displaystyle\int_{3}^{11} \dfrac{x}{\sqrt{x-2}}\, dx = 25\tfrac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**

9   Fig. 9 shows the curves $y = f(x)$ and $y = g(x)$. The function $y = f(x)$ is given by

$$f(x) = \ln\left(\frac{2x}{1+x}\right), \quad x > 0.$$

The curve $y = f(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.
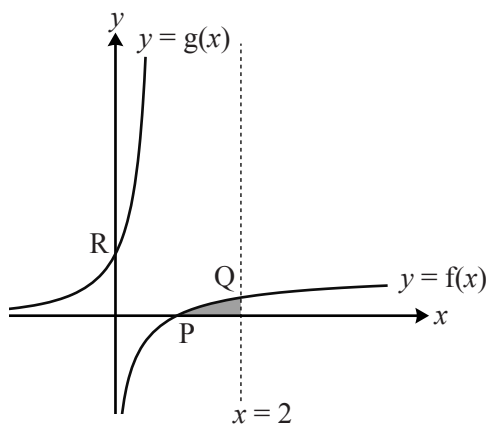
Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln\dfrac{a}{b} = \ln a - \ln b$.] **[4]**

The function $g(x)$ is given by

$$g(x) = \frac{e^x}{2 - e^x}, \quad x < \ln 2.$$

The curve $y = g(x)$ crosses the y-axis at the point R.

(iii) Show that $g(x)$ is the inverse function of $f(x)$.

Write down the gradient of $y = g(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - e^x$ or otherwise, that $\displaystyle\int_0^{\ln\frac{4}{3}} g(x)\,dx = \ln\frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln\frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**

![OCR logo] RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01   Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
* Printed Answer Book 4753/01
* MEI Examination Formulae and Tables (MF2)

Other materials required:
* Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Turn over

Section A (36 marks)

1   Show that $\displaystyle\int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \frac{2}{3}$. **[5]**

2   Solve the inequality $|2x+1| > 4$. **[3]**

3   Find the gradient at the point $(0, \ln 2)$ on the curve with equation $e^{2y} = 5 - e^{-x}$. **[4]**

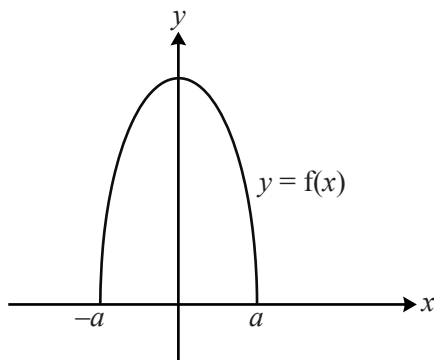4   Fig. 4 shows the curve $y = f(x)$, where $f(x) = \sqrt{1-9x^2}$, $-a \leqslant x \leqslant a$.

Fig. 4

   (i) Find the value of a. **[2]**

   (ii) Write down the range of $f(x)$. **[1]**

   (iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. **[3]**

5   A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

   (i) Calculate the population when $t = 0$, and the long-term population, given by this model. **[3]**

   (ii) Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k. **[3]**

6 Fig. 6 shows the curve $y = f(x)$, where $f(x) = 2\arcsin x$, $-1 \leqslant x \leqslant 1$.

Fig. 6 also shows the curve $y = g(x)$, where $g(x)$ is the inverse function of $f(x)$.

P is the point on the curve $y = f(x)$ with x-coordinate $\frac{1}{2}$.
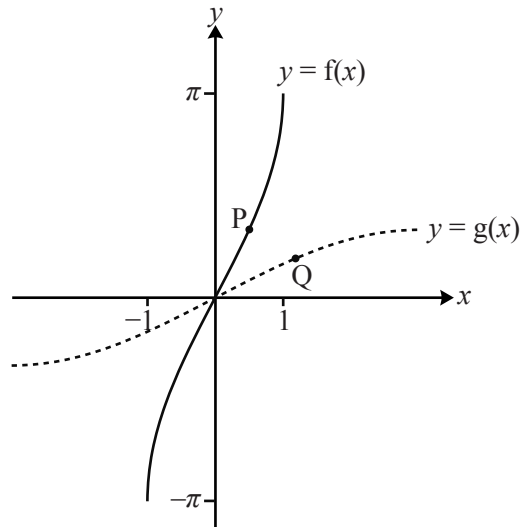
Fig. 6

   (i) Find the y-coordinate of P, giving your answer in terms of π.   **[2]**

The point Q is the reflection of P in $y = x$.

   (ii) Find $g(x)$ and its derivative $g'(x)$. Hence determine the exact gradient of the curve $y = g(x)$ at the point Q.

      Write down the exact gradient of $y = f(x)$ at the point P.   **[6]**

7 You are given that $f(x)$ and $g(x)$ are odd functions, defined for $x \in \mathbb{R}$.

   (i) Given that $s(x) = f(x) + g(x)$, prove that $s(x)$ is an odd function.   **[2]**

   (ii) Given that $p(x) = f(x)g(x)$, determine whether $p(x)$ is odd, even or neither.   **[2]**

4

8 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.
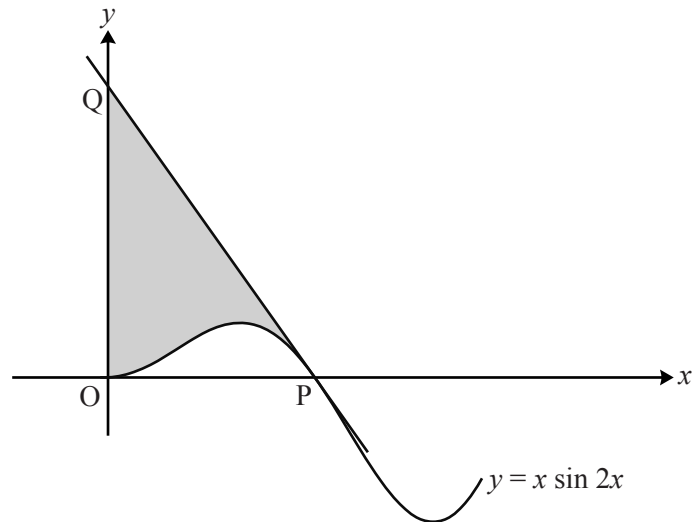
Fig. 8

(i) Find $\dfrac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**

9 Fig. 9 shows the curve $y = f(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.
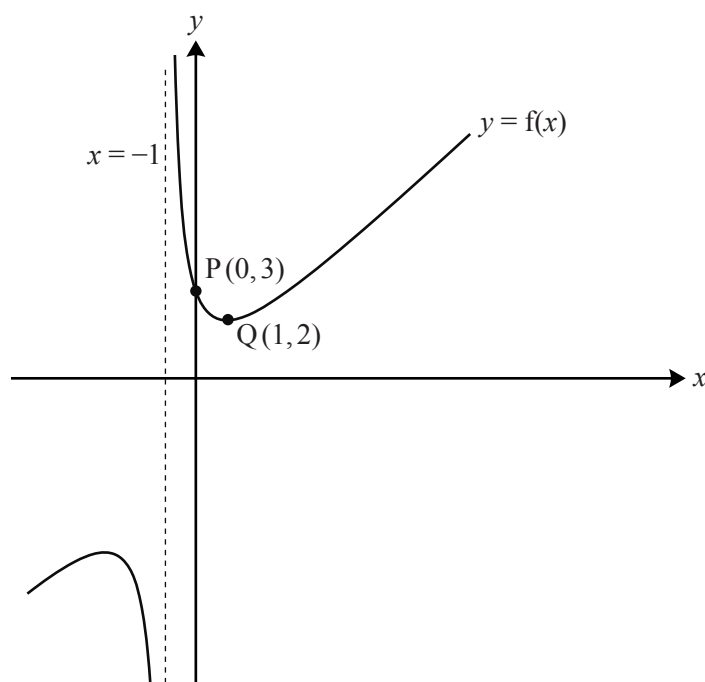
Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to

(A) $y = 2f(x)$,

(B) $y = f(x + 1) + 2$. **[4]**

You are now given that $f(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$. **[6]**

(iii) Show that $f(x - 1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \dfrac{4}{x}\right) dx$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Given that $y = \mathrm{e}^{-x} \sin 2x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[3]**

(ii) Hence show that the curve $y = \mathrm{e}^{-x} \sin 2x$ has a stationary point when $x = \frac{1}{2} \arctan 2$. **[3]**

2 A curve has equation $x^2 + 2y^2 = 4x$.

(i) By differentiating implicitly, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[3]**

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] **[3]**

3 Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. **[2]**

4 The temperature $\theta\,^{\circ}\mathrm{C}$ of water in a container after t minutes is modelled by the equation

$$\theta = a - b\mathrm{e}^{-kt},$$

where a, b and k are positive constants.

The initial and long-term temperatures of the water are $15\,^{\circ}\mathrm{C}$ and $100\,^{\circ}\mathrm{C}$ respectively. After 1 minute, the temperature is $30\,^{\circ}\mathrm{C}$.

(i) Find a, b and k. **[6]**

(ii) Find how long it takes for the temperature to reach $80\,^{\circ}\mathrm{C}$. **[2]**

5 The driving force F newtons and velocity $v\ \mathrm{km\,s^{-1}}$ of a car at time t seconds are related by the equation $F = \dfrac{25}{v}$.

(i) Find $\dfrac{\mathrm{d}F}{\mathrm{d}v}$. **[2]**

(ii) Find $\dfrac{\mathrm{d}F}{\mathrm{d}t}$ when $v = 50$ and $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 1.5$. **[3]**

6 Evaluate $\displaystyle\int_0^3 x(x + 1)^{-\frac{1}{2}}\,\mathrm{d}x$, giving your answer as an exact fraction. **[5]**

7 **(i)** Disprove the following statement:

$$3^n + 2 \text{ is prime for all integers } n \geqslant 0.$$ **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

Section B (36 marks)

8 Fig. 8 shows parts of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = \tan x$ and $g(x) = 1 + f(x - \frac{1}{4}\pi)$.
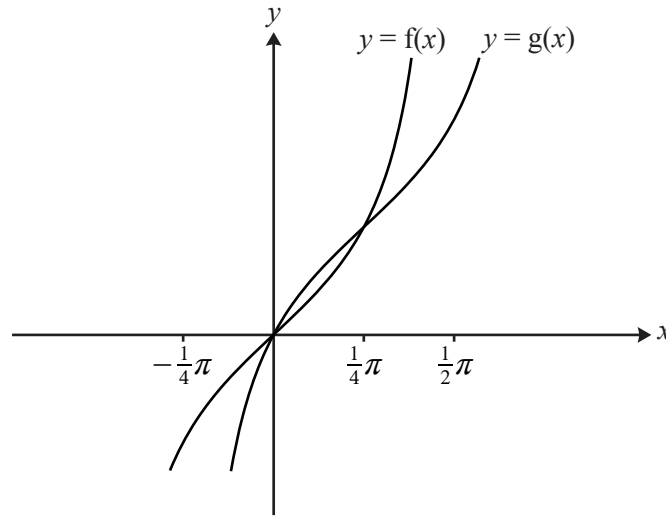
Fig. 8

  (i) Describe a sequence of two transformations which maps the curve $y = f(x)$ to the curve $y = g(x)$. **[4]**

It can be shown that $g(x) = \dfrac{2\sin x}{\sin x + \cos x}$.

  (ii) Show that $g'(x) = \dfrac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = g(x)$ at the point $(\frac{1}{4}\pi, 1)$ is

    the same as that of $y = f(x)$ at the origin. **[7]**

  (iii) By writing $\tan x = \dfrac{\sin x}{\cos x}$ and using the substitution $u = \cos x$, show that $\displaystyle\int_0^{\frac{1}{4}\pi} f(x)\,dx = \int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{u}\,du$.

    Evaluate this integral exactly. **[4]**

  (iv) Hence find the exact area of the region enclosed by the curve $y = g(x)$, the x-axis and the lines
    $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$. **[2]**

9 Fig. 9 shows the line $y = x$ and the curve $y = f(x)$, where $f(x) = \frac{1}{2}(e^x - 1)$. The line and the curve intersect at the origin and at the point P(a, a).
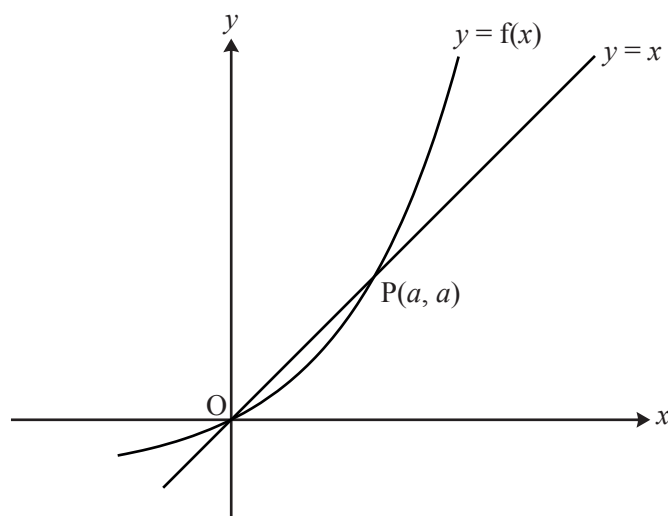
Fig. 9

   (i) Show that $e^a = 1 + 2a$. **[1]**

   (ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

   (iii) Show that the inverse function of $f(x)$ is $g(x)$, where $g(x) = \ln(1 + 2x)$. Add a sketch of $y = g(x)$ to the copy of Fig. 9. **[5]**

   (iv) Find the derivatives of $f(x)$ and $g(x)$. Hence verify that $g'(a) = \dfrac{1}{f'(a)}$.

   Give a geometrical interpretation of this result. **[7]**

OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   The Question Paper will be found in the centre of the Printed Answer Book.
*   Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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*   Answer **all** the questions.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1   Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x- and y-axes are $(-1, 0)$ and $\left(0, \frac{1}{2}\right)$ respectively.
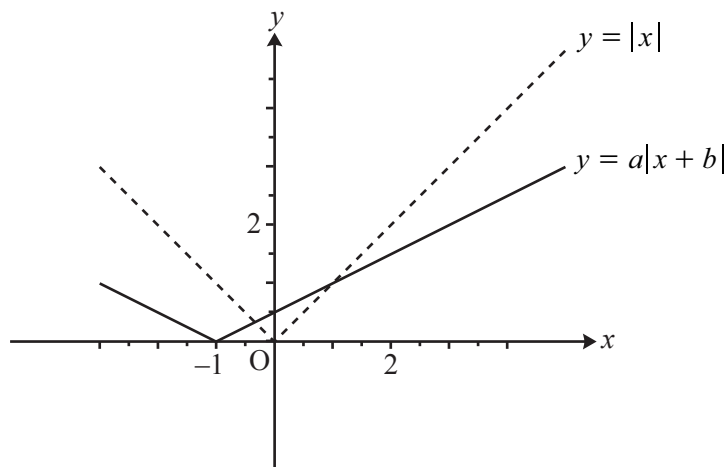

Fig. 1

(i) Find a and b. [2]

(ii) Find the coordinates of the two points of intersection of the graphs. [4]

2   (i) Factorise fully $n^3 - n$. [2]

(ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. [2]

3 The function f(x) is defined by $f(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.
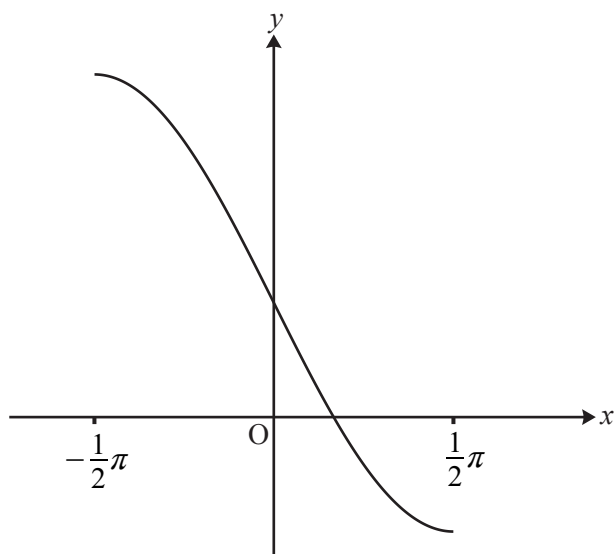
Fig. 3

   (i) Write down the range of the function f(x). **[2]**

   (ii) Find the inverse function $f^{-1}(x)$. **[3]**

   (iii) Find f′(0). Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. **[3]**

4 Water flows into a bowl at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$ (see Fig. 4).
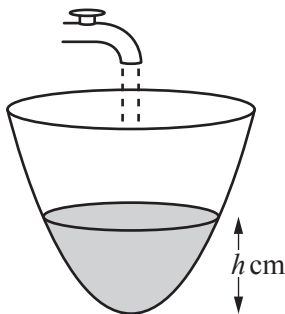
Fig. 4

When the depth of water in the bowl is $h\,\text{cm}$, the volume of water is $V\,\text{cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. **[5]**

5 Given that $y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$, show that $\dfrac{dy}{dx} = \dfrac{1}{2x-1} - \dfrac{1}{2x+1}$. **[4]**

6 Using a suitable substitution or otherwise, show that $\displaystyle\int_0^{\frac{1}{2}\pi} \dfrac{\sin 2x}{3 + \cos 2x}\,dx = \dfrac{1}{2}\ln 2$. **[5]**

7 **(i)** Show algebraically that the function $f(x) = \dfrac{2x}{1 - x^2}$ is odd. **[2]**

Fig. 7 shows the curve $y = f(x)$ for $0 \leqslant x \leqslant 4$, together with the asymptote $x = 1$.
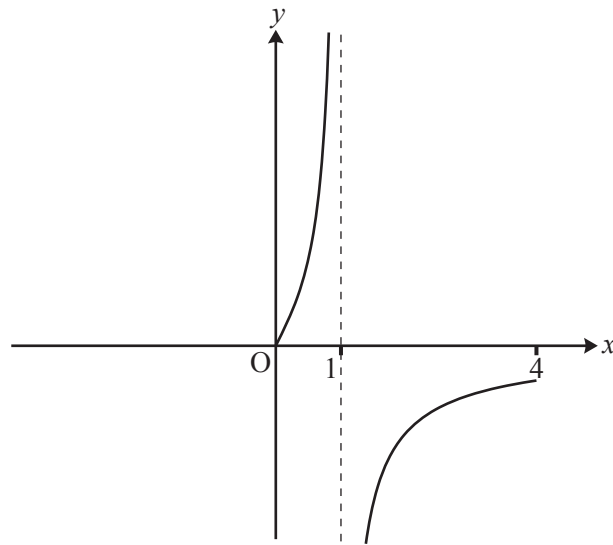
Fig. 7

(ii) Use the copy of Fig. 7 to complete the curve for $-4 \leqslant x \leqslant 4$. **[2]**

Section B (36 marks)

8   Fig. 8 shows the curve $y = f(x)$, where $f(x) = (1 - x)e^{2x}$, with its turning point P.
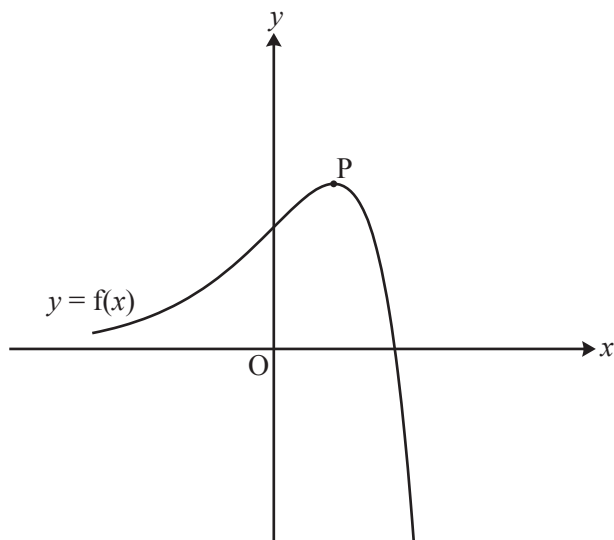
Fig. 8

(i) Write down the coordinates of the intercepts of $y = f(x)$ with the x- and y-axes.   **[2]**

(ii) Find the exact coordinates of the turning point P.   **[6]**

(iii) Show that the exact area of the region enclosed by the curve and the x- and y-axes is $\frac{1}{4}(e^2 - 3)$.   **[5]**

The function $g(x)$ is defined by $g(x) = 3f\left(\frac{1}{2}x\right)$.

(iv) Express $g(x)$ in terms of x.

Sketch the curve $y = g(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x- and y-axes and of its turning point.   **[4]**

(v) Write down the exact area of the region enclosed by the curve $y = g(x)$ and the x- and y-axes.   **[1]**

9 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.
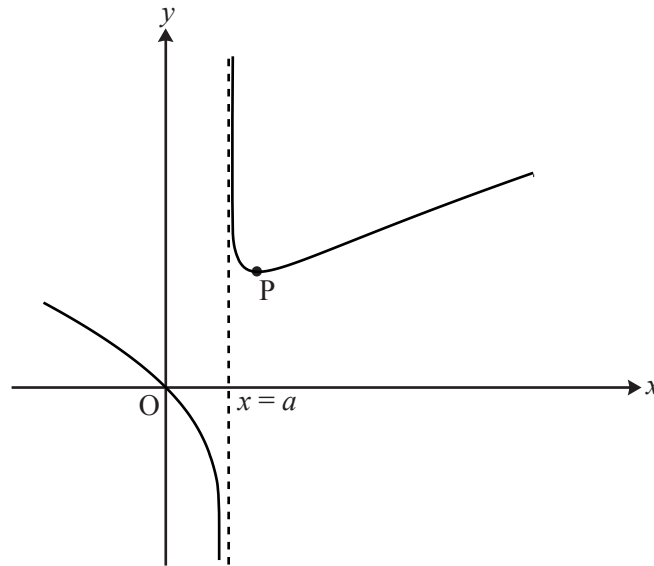
Fig. 9

 (i) Write down the value of a. **[1]**

 (ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

   Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

 (iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \dfrac{x}{\sqrt[3]{2x-1}}\,\mathrm{d}x$ to $\dfrac{1}{4}\displaystyle\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right)\mathrm{d}u$.

   Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

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Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01   Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
*   Printed Answer Book 4753/01
*   MEI Examination Formulae and Tables (MF2)

Other materials required:
*   Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

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*   Answer **all** the questions.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1   Evaluate $\int_0^{\frac{1}{6}\pi}(1-\sin 3x)\,\mathrm{d}x$, giving your answer in exact form.   **[3]**

2   Find the exact gradient of the curve $y=\ln(1-\cos 2x)$ at the point with x-coordinate $\frac{1}{6}\pi$.   **[5]**

3   Solve the equation $|3-2x|=4|x|$.   **[4]**

4   Fig. 4 shows the curve $y=\mathrm{f}(x)$, where

$$\mathrm{f}(x)=a+\cos bx,\ 0\leqslant x\leqslant 2\pi,$$

and a and b are positive constants. The curve has stationary points at $(0,3)$ and $(2\pi,1)$.
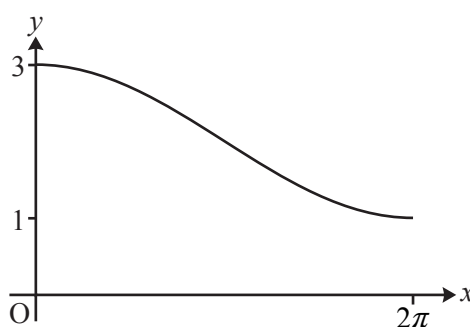
Fig. 4

   (i) Find a and b.   **[2]**

   (ii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range.   **[5]**

5   A spherical balloon of radius r cm has volume $V\,\mathrm{cm}^3$, where $V=\frac{4}{3}\pi r^3$. The balloon is inflated at a constant rate of $10\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Find the rate of increase of r when $r=8$.   **[5]**

6   The value $£V$ of a car t years after it is new is modelled by the equation $V=A\mathrm{e}^{-kt}$, where A and k are positive constants which depend on the make and model of the car.

   (i) Brian buys a new sports car. Its value is modelled by the equation

$$V=20\,000\,\mathrm{e}^{-0.2t}.$$

   Calculate how much value, to the nearest £100, this car has lost after 1 year.   **[2]**

   (ii) At the same time as Brian buys his car, Kate buys a new hatchback for £15000. Her car loses £2000 of its value in the first year. Show that, for Kate's car, $k=0.143$ correct to 3 significant figures.   **[3]**

   (iii) Find how long it is before Brian's and Kate's cars have the same value.   **[3]**

7   Either prove or disprove each of the following statements.

   (i) 'If m and n are consecutive odd numbers, then at least one of m and n is a prime number.'   **[2]**

   (ii) 'If m and n are consecutive even numbers, then mn is divisible by 8.'   **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{x}{\sqrt{2+x^2}}$.
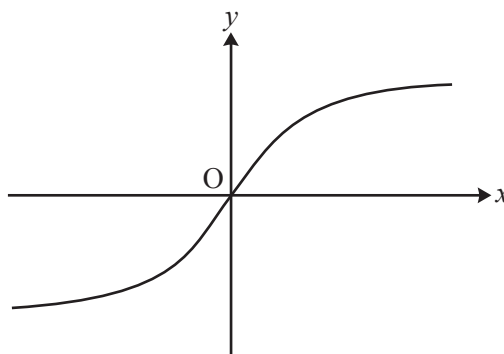

Fig. 8

(i) Show algebraically that $f(x)$ is an odd function. Interpret this result geometrically. **[3]**

(ii) Show that $f'(x) = \dfrac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin. **[5]**

(iii) Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$. **[4]**

(iv) (*A*) Show that if $y = \dfrac{x}{\sqrt{2+x^2}}$, then $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$. **[2]**

(*B*) Differentiate $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$ implicitly to show that $\dfrac{dy}{dx} = \dfrac{2y^3}{x^3}$. Explain why this expression cannot

be used to find the gradient of the curve at the origin. **[4]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = x\mathrm{e}^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.
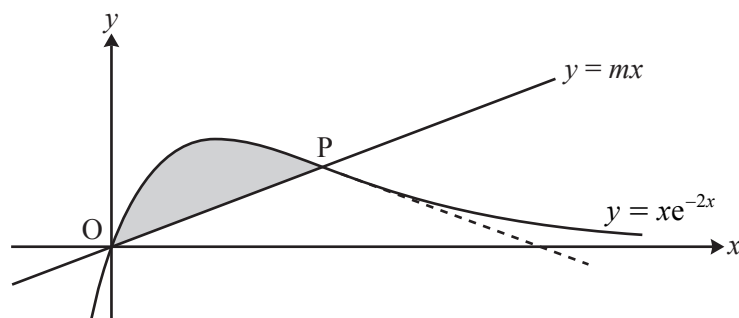
Fig. 9

   (i) Show that the x-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

   (ii) Find, in terms of m, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the x-axis.

   (iii) Show that $m = \mathrm{e}^{-2}$, and find the exact coordinates of P. **[4]**

   (iv) Find the exact area of the shaded region between the line OP and the curve. **[7]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01   Methods for Advanced Mathematics (C3)

QUESTION PAPER

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Turn over

Section A (36 marks)

1   Fig. 1 shows part of the curve $y = e^{2x} \cos x$.

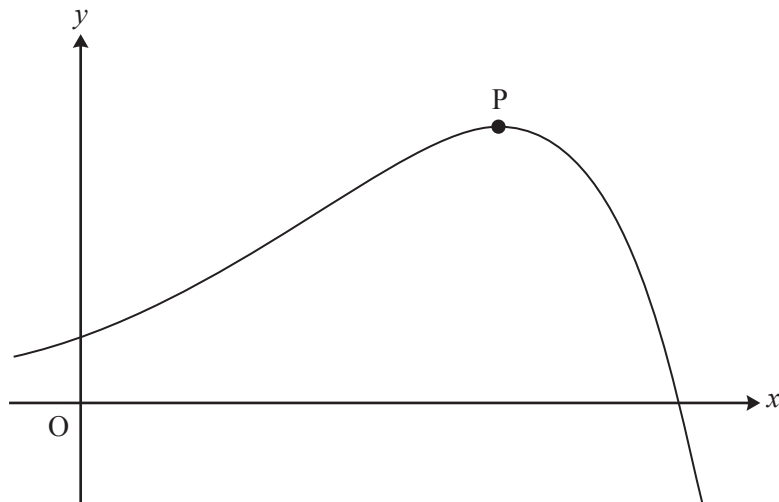**Fig. 1**

Find the coordinates of the turning point P.   **[6]**

2   Find $\displaystyle\int \sqrt[3]{2x-1}\, dx$.   **[4]**

3   Find the exact value of $\displaystyle\int_1^2 x^3 \ln x\, dx$.   **[5]**

4   Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of $5\,cm^3$ per second. At time t seconds, the height of the water surface above the vertex O of the cone is $h\,cm$, and the volume of water in the cone is $V\,cm^3$.
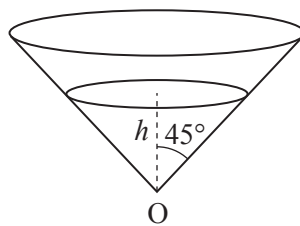
Fig. 4

Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is $10\,cm$.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$].   **[5]**

5   A curve has implicit equation $y^2 + 2x \ln y = x^2$.

Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point.   **[6]**

6   Solve each of the following equations, giving your answers in exact form.

   (i) $6\arcsin x - \pi = 0$. **[2]**

   (ii) $\arcsin x = \arccos x$. **[2]**

7   **(i)** The function f(x) is defined by

$$\mathrm{f}(x) = \frac{1-x}{1+x}, \quad x \neq -1.$$

   Show that $\mathrm{f}(\mathrm{f}(x)) = x$.

   Hence write down $\mathrm{f}^{-1}(x)$. **[3]**

   (ii) The function g(x) is defined for all real x by

$$\mathrm{g}(x) = \frac{1-x^2}{1+x^2}.$$

   Prove that g(x) is even. Interpret this result in terms of the graph of $y = \mathrm{g}(x)$. **[3]**

Section B (36 marks)

8   Fig. 8 shows the line $y = 1$ and the curve $y = f(x)$, where $f(x) = \dfrac{(x-2)^2}{x}$. The curve touches the x-axis at P(2, 0) and has another turning point at the point Q.
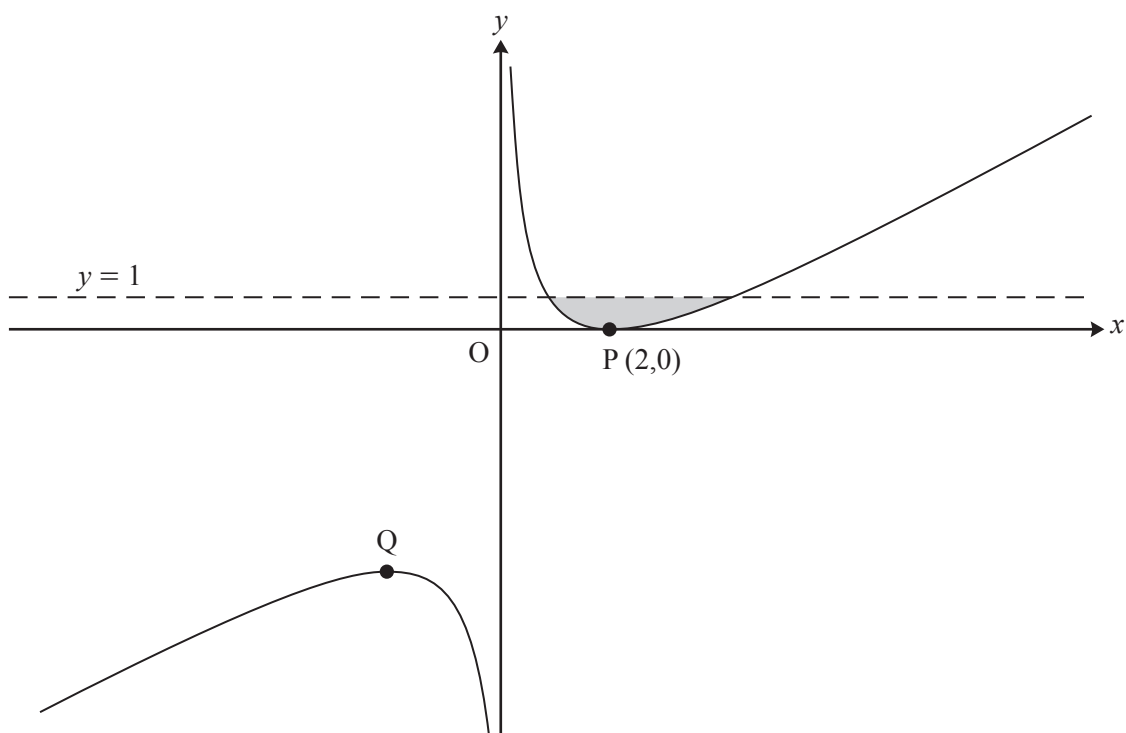
Fig. 8

  (i) Show that $f'(x) = 1 - \dfrac{4}{x^2}$, and find $f''(x)$.

Hence find the coordinates of Q and, using $f''(x)$, verify that it is a maximum point.   **[7]**

  (ii) Verify that the line $y = 1$ meets the curve $y = f(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve.   **[6]**

The curve $y = f(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = g(x)$.

  (iii) Show that $g(x) = \dfrac{x^2 - 3x}{x+1}$.   **[3]**

  (iv) Without further calculation, write down the value of $\int_0^3 g(x)\,dx$, justifying your answer.   **[2]**

9 Fig. 9 shows the curve $y = f(x)$, where

$$f(x) = (e^x - 2)^2 - 1, \ x \in \mathbb{R}.$$

The curve crosses the x-axis at O and P, and has a turning point at Q.
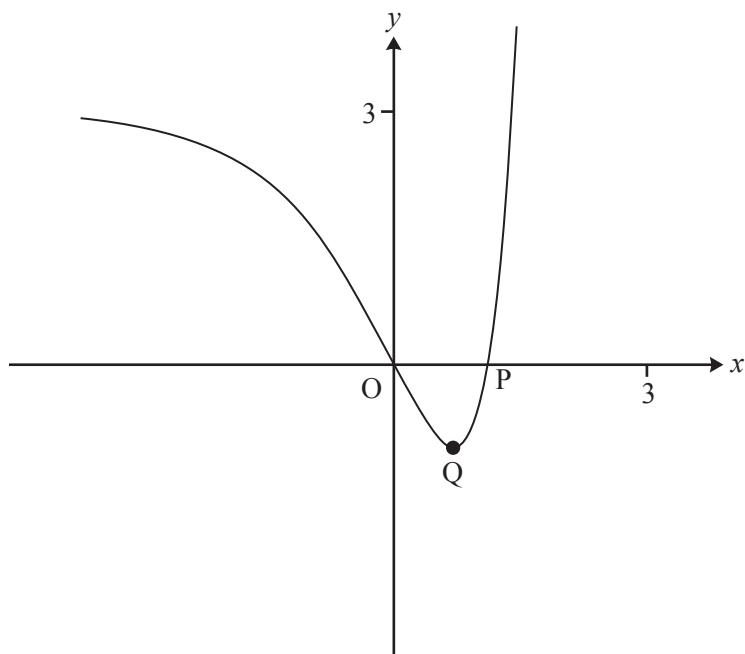
Fig. 9

   (i) Find the exact x-coordinate of P. **[2]**

  (ii) Show that the x-coordinate of Q is $\ln 2$ and find its y-coordinate. **[4]**

 (iii) Find the exact area of the region enclosed by the curve and the x-axis. **[5]**

The domain of $f(x)$ is now restricted to $x \geqslant \ln 2$.

 (iv) Find the inverse function $f^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. **[7]**

END OF QUESTION PAPER

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Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01   Methods for Advanced Mathematics (C3)

QUESTION PAPER

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

• The Question Paper will be found inside the Printed Answer Book.
• Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
• **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
• Use black ink. HB pencil may be used for graphs and diagrams only.
• Read each question carefully. Make sure you know what you have to do before starting your answer.
• Answer **all** the questions.
• Do **not** write in the bar codes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

• The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
• You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
• The total number of marks for this paper is **72**.
• The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1   Find the exact value of $\int_0^{\frac{1}{2}\pi}(1+\cos\frac{1}{2}x)\,dx$. [3]

2   The functions f(x) and g(x) are defined by f(x) = ln x and g(x) = 2 + e^x, for $x > 0$.

    Find the exact value of x, given that fg(x) = 2x. [5]

3   Find $\int_1^4 x^{-\frac{1}{2}}\ln x\,dx$, giving your answer in an exact form. [5]

4   By sketching the graphs of $y = |2x+1|$ and $y = -x$ on the same axes, show that the equation $|2x+1| = -x$ has two roots. Find these roots. [4]

5   The volume $V\,\text{m}^3$ of a pile of grain of height h metres is modelled by the equation

    $$V = 4\sqrt{h^3+1}-4.$$

    (i) Find $\dfrac{dV}{dh}$ when $h = 2$. [4]

    At a certain time, the height of the pile is 2 metres, and grain is being added so that the volume is increasing at a rate of $0.4\,\text{m}^3$ per minute.

    (ii) Find the rate at which the height is increasing at this time. [3]

6   Fig. 6 shows part of the curve $\sin 2y = x - 1$. P is the point with coordinates $(1.5, \frac{1}{12}\pi)$ on the curve.
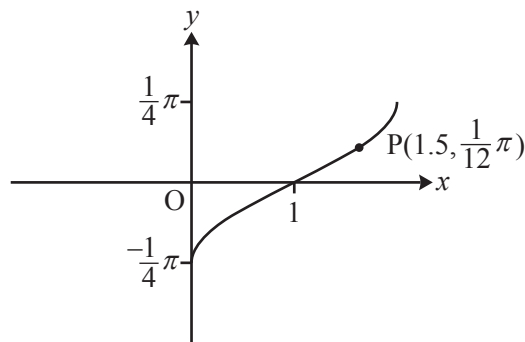

Fig. 6

    (i) Find $\dfrac{dy}{dx}$ in terms of y.

    Hence find the exact gradient of the curve $\sin 2y = x - 1$ at the point P. [4]

    The part of the curve shown is the image of the curve $y = \arcsin x$ under a sequence of two geometrical transformations.

    (ii) Find y in terms of x for the curve $\sin 2y = x - 1$.

    Hence describe fully the sequence of transformations. [4]

7 You are given that *n* is a positive integer.

By expressing $x^{2n} - 1$ as a product of two factors, prove that $2^{2n} - 1$ is divisible by 3.   **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x+4}}$ and the line $x = 5$. The curve has an asymptote *l*.

The tangent to the curve at the origin O crosses the line *l* at P and the line $x = 5$ at Q.
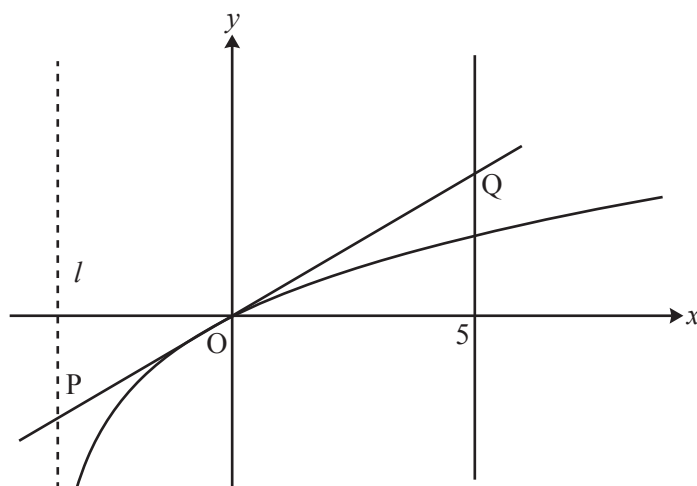

Fig. 8

(i) Show that for this curve $\dfrac{dy}{dx} = \dfrac{x+8}{2(x+4)^{\frac{3}{2}}}$.   **[5]**

(ii) Find the coordinates of the point P.   **[4]**

(iii) Using integration by substitution, find the exact area of the region enclosed by the curve, the tangent OQ and the line $x = 5$.   **[9]**

9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = e^{2x} + k\,e^{-2x}$ and k is a constant greater than 1.

The curve crosses the y-axis at P and has a turning point Q.
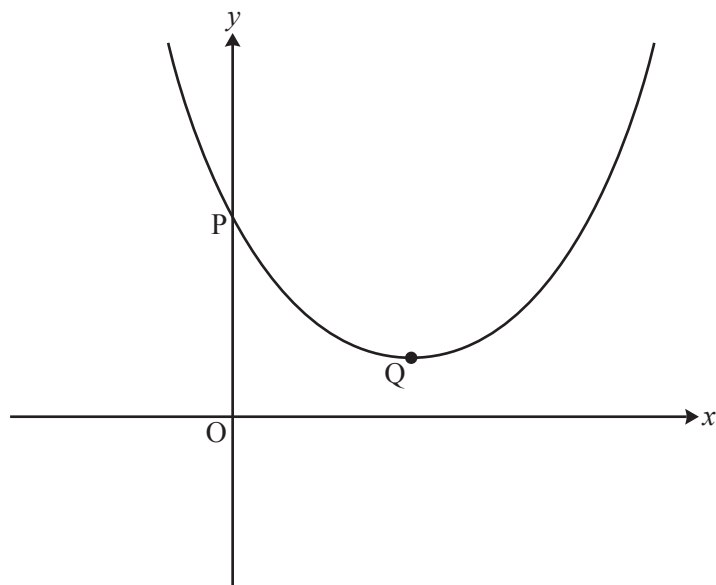
Fig. 9

(i) Find the y-coordinate of P in terms of k. **[1]**

(ii) Show that the x-coordinate of Q is $\frac{1}{4}\ln k$, and find the y-coordinate in its simplest form. **[5]**

(iii) Find, in terms of k, the area of the region enclosed by the curve, the x-axis, the y-axis and the line $x = \frac{1}{2}\ln k$. Give your answer in the form $ak + b$. **[4]**

The function $g(x)$ is defined by $g(x) = f(x + \frac{1}{4}\ln k)$.

(iv) (*A*) Show that $g(x) = \sqrt{k}\,(e^{2x} + e^{-2x})$. **[3]**

(*B*) Hence show that $g(x)$ is an even function. **[2]**

(*C*) Deduce, with reasons, a geometrical property of the curve $y = f(x)$. **[3]**

END OF QUESTION PAPER